KPK BOARD NOTES

# MATHEMATICS

# 9TH CLASS

Presented by:

Urdu Books Whatsapp Group

## STUDY GROUP


0333-8033313
راؤ ایاز

0343-7008883
پاکستان زندہ باد

0306-7163117
محمد سلمان سلیم

Tehkals.com
Learn & Teach

# MATHEMATICS

## Class 9th (KPK)

NAME: ______________________________

F.NAME: _____________________________

CLASS:_____________ SECTION: _________

ROLL #: ______ SUBJECT: ______________

ADDRESS: _________________________________________

_________________________________________________

SCHOOL: __________________________________________

## *Ex # 2.1*

Set of Natural Numbers

$$N = \{1, 2, 3, 4, \ldots\}$$

Set of Whole Numbers

$$W = \{0, 1, 2, 3, 4, \ldots\}$$

Set of Integers

$$Z = \{0, \pm1, \pm2, \pm3, \ldots\}$$

OR

$$Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3\}$$

**Rational Numbers**

The word Rational means "Ratio".

A rational number is a number that can be expressed in the form of $\frac{p}{q}$ where p and q are integers and $q \neq 0$. Rational numbers is denoted by $Q$.

**Set of Rational Numbers**

$$Q = \left\{\frac{p}{q} \mid p, q \in Z, q \neq 0\right\}$$

**Irrational Numbers**

The word Irrational means "Not Ratio".

Irrational number consists of all those numbers which are not rational. Irrational numbers is denoted by $Q^{/}$.

**Real numbers**

The set of rational and irrational numbers is called Real Numbers. Real numbers is denoted by $R$.

Thus $Q \cup Q^{/} = R$

**Note:**

All the numbers on the number line are real numbers.

**Terminating Decimal Fraction:**

A decimal number that contains a finite number of digits after the decimal point.

Non-Terminating Decimal Fraction:

A decimal number that has no end after the decimal point.

**Non-Terminating Repeating Decimal Fraction**

In non-terminating decimal fraction, some digits are repeated in same order after decimal point.

**Non-Terminating Non-Repeating Decimal Fraction.**

In non-terminating decimal fraction, the digits are not repeated in same order after decimal point.

**Decimal Representation of Rational and Irrational Numbers.**

**(i)** All terminating and repeating decimals are rational numbers.

**(ii)** Non-terminating recurring (repeating) decimals are rational numbers.

**(iii)** Never terminating or repeating decimals are irrational numbers.

Non-terminating and non-recurring (repeating) decimals are irrational numbers.

**Note:**

**(i)** Repeating decimals are called recurring decimals.

**(ii)** Non-repeating decimals are called non-recurring decimals.

ختم نبوت ﷺ زندہ باد

عظمت صحابہ زندہ باد

**السلام علیکم ورحمۃ اللہ وبرکاتہ:**

معزز ممبران: آپ کا وٹس ایپ گروپ ایڈ من "**اردو بکس**" آپ سے مخاطب ہے۔

**آپ تمام ممبران سے گزارش ہے کہ:**

❖ گروپ میں صرف PDF کتب پوسٹ کی جاتی ہیں لہذا کتب کے متعلق اپنے کمنٹس / ریویوز ضرور دیں۔ گروپ میں بغیر ایڈ من کی اجازت کے کسی بھی قسم کی (اسلامی وغیر اسلامی، اخلاقی، تحریری) پوسٹ کرنا سختی سے منع ہے۔

❖ گروپ میں معزز، پڑھے لکھے، سلجھے ہوئے ممبرز موجود ہیں اخلاقیات کی پابندی کریں اور گروپ رولز کو فالو کریں بصورت دیگر معزز ممبرز کی بہتری کی خاطر ریموو کر دیا جائے گا۔

❖ کوئی بھی ممبر کسی بھی ممبر کو انباکس میں میسج، مس کال، کال نہیں کرے گا۔ رپورٹ پر فوری ریموو کر کے کاروائی عمل میں لائے جائے گی۔

❖ ہمارے کسی بھی گروپ میں سیاسی و فرقہ واریت کی بحث کی قطعاً کوئی گنجائش نہیں ہے۔

❖ اگر کسی کو بھی گروپ کے متعلق کسی قسم کی شکایت یا تجویز کی صورت میں ایڈ من سے رابطہ کیجئے۔

❖ سب سے اہم بات:

**گروپ میں کسی بھی قادیانی، مرزائی، احمدی، گستاخِ رسول، گستاخِ امہات المؤمنین، گستاخِ صحابہ و خلفائے راشدین حضرت ابو بکر صدیق، حضرت عمر فاروق، حضرت عثمان غنی، حضرت علی المرتضیٰ، حضرت حسنین کریمین رضوان اللہ تعالیٰ اجمعین، گستاخ اہلبیت یا ایسے غیر مسلم جو اسلام اور پاکستان کے خلاف پراپیگنڈا میں مصروف ہیں یا ان کے روحانی و ذہنی سپورٹرز کے لئے کوئی گنجائش نہیں ہے لہذا ایسے اشخاص بالکل بھی گروپ جوائن کرنے کی زحمت نہ کریں۔ معلوم ہونے پر فوراً ریموو کر دیا جائے گا۔**

❖ تمام کتب انٹرنیٹ سے تلاش / ڈاؤنلوڈ کر کے **فری آف کاسٹ** وٹس ایپ گروپ میں شیئر کی جاتی ہیں۔ جو کتاب نہیں ملتی اس کے لئے معذرت کر لی جاتی ہے۔ جس میں محنت بھی صَرف ہوتی ہے لیکن ہمیں آپ سے صرف دعاؤں کی درخواست ہے۔

❖ عمران سیریز کے شوقین کیلئے علیحدہ سے عمران سیریز گروپ موجود ہے۔

❖ **لیڈیز کے لئے الگ گروپ کی سہولت موجود ہے جس کے لئے ویریفکیشن ضروری ہے۔**

❖ اردو کتب / عمران سیریز یا سٹڈی گروپ میں ایڈ ہونے کے لئے ایڈ من سے وٹس ایپ پر بذریعہ میسج رابطہ کریں اور جواب کا انتظار فرمائیں۔ برائے مہربانی اخلاقیات کا خیال رکھتے ہوئے موبائل پر کال یا ایم ایس کرنے کی کوشش ہر گز نہ کریں۔ ورنہ گروپس سے تو ریموو کیا ہی جائے گا بلاک بھی کیا جائے گا۔

**نوٹ: ہمارے کسی گروپ کی کوئی فیس نہیں ہے۔ سب فی سبیل اللہ ہے**

| 0306-7163117 | 0343-7008883 | 0333-8033313 |
|---|---|---|
| محمد سلمان سلیم | پاکستان زندہ باد | راؤ ایاز |

پاکستان زندہ باد

پاکستان پائندہ باد

اللہ تبارک تعالیٰ ہم سب کا حامی وناصر ہو

## ***Ex # 2.1***
## ***Page # 54***

**In Questions 1 – 10, consider the numbers.**

$$\mathbf{2.5, 3, \frac{5}{7}, -1.96, 0, \sqrt{36}, -\frac{7}{6}, \sqrt{3}, -9, 1, \sqrt{7}, -\sqrt{14}, \pi, 4\frac{2}{3}, 0.333\ldots}$$

**1. Which are whole numbers?**

Ans: $3, \quad 0, \quad \sqrt{36}, \quad 1$

**2. Which are integers?**

Ans: $3, \quad 0, \quad \sqrt{36}, \quad -9, \quad 1$

**3. Which are irrational numbers?**

Ans: $\sqrt{3}, \quad \sqrt{7}, -\sqrt{14}, \quad \pi$

**4. Which are natural numbers?**

Ans: $3, \quad \sqrt{36}, \quad 1$

**5. Which are rational numbers?**

Ans: $2.5, 3, \frac{5}{7}, -1.96, 0, \sqrt{36}, -\frac{7}{6}, -9, 1, \quad 4\frac{2}{3},$ $0.333\ldots$

**6. Which are real numbers?**

Ans: $2.5, 3, \frac{5}{7}, -1.96, 0, \sqrt{36}, -\frac{7}{6}, \sqrt{3}, -9, 1, \sqrt{7}, -\sqrt{14}, \pi, 4\frac{2}{3}, 0.333.$

**7. Which are rational numbers but not integers?**

Ans: $2.5, \quad \frac{5}{7}, -1.96, \quad -\frac{7}{6}, \quad 4\frac{2}{3}, \quad 0.333\ldots$

**8. Which are integers but not whole numbers?**

Ans: $-9$

**9. Which are integers but not natural numbers?**

Ans: $0, -9$

**10. Which are real numbers but not integers?**

Ans: $2.5, \quad \frac{5}{7}, -1.96, \quad -\frac{7}{6}, \quad \sqrt{3}, \quad \sqrt{7}, -\sqrt{14}, \quad \pi,$ $4\frac{2}{3}, \quad 0.333\ldots$



**11. Write the decimal representation of each of the following numbers.**

$\frac{1}{6}, \frac{6}{7}, \frac{2}{9}, \frac{1}{8}$

$\frac{1}{6} = 0.1666\ldots.$

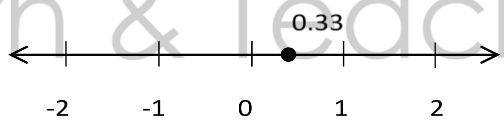
$\frac{6}{7} = 0.8571\ldots.$

$\frac{2}{9} = 0.222\ldots.$

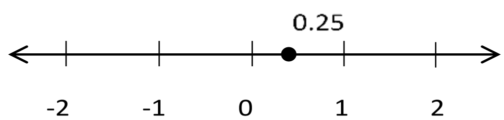
$\frac{1}{8} = 0.125$

**12 Depict each number on a number line.**

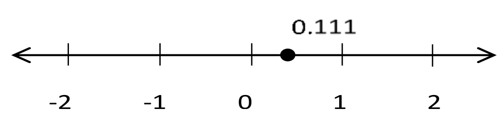
**(i)** $\mathbf{\frac{1}{3} = 0.333\ldots}$



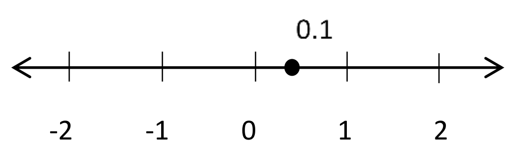

**(ii)** $\mathbf{\frac{1}{4} = 0.25}$




**(ii)** $\frac{1}{9} = 0.111\ldots$




**(iv)** $\frac{1}{10} = 0.1$



## Ex # 2.2

**Properties of Real Number**

The set R of real number is the union of two disjoint sets. Thus $R = Q \cup Q^{/}$

**Note:**

$$Q \cap Q^{/} = \emptyset$$

**Real Number System**

Closure Property w.r.t Addition

The sum of real number is also a real number.

If $a,\ b\ \in R$ then $a + b\ \in R$

**Example:**

$$7 + 9 = 16$$

Where 16 is a real number.

**Closure Property w.r.t Multiplication**

The Product of real number is also a real number.

If $a,\ b\ \in R$ then $a\,.\,b\ \in R$

**Example:**

$$7 \times 9 = 63$$

Where 63 is a real number.

**Commutative Property w.r.t Addition**

If $a,\ b\ \in R$ then $a + b = b + a$

**Example:**

$$7 + 9 = 9 + 7$$
$$16 = 16$$

**Commutative Property w.r.t Multiplication**

If $a,\ b\ \in R$ then $a\,.\ b = b\,.\ a$

**Example:**

$$7 \times 9 = 9 \times 7$$
$$63 = 63$$

**Associative Property w.r.t Addition**

If $a,\ b,\ c\ \in R$ then

$$a + (b + c) = (a + b) + c$$

**Example:**

$$2 + (3 + 5) = (2 + 3) + 5$$
$$2 + 8 = 5 + 5$$
$$10 = 10$$

**Associative Property w.r.t Multiplication**

If $a,\ b,\ c\ \in R$ then

$$a(bc) = (ab)c$$

**Example:**

$$2(3 \times 5) = (2 \times 3)5$$
$$2(15) = (6)5$$
$$30 = 30$$

**Additive Identity**

Zero (0) is called Additive identity because adding "0" to a number does not change that number.

If $a \in R$ there exists $O\ \in R$ then

$$a + 0 = 0 + a = a$$

**Example:**

$$3 + 0 = 0 + 3 = 3$$

**Multiplicative Identity**

1 is called Multiplicative identity because multiplying "1" to a number does not change that number.

If $a \in R$ there exists $1\ \in R$ then

$$a\,.\,1 = 1\,.\,a = a$$

**Example:**

$$3 \times 1 = 1 \times 3 = 3$$

**Additive Inverse**

When the sum of two numbers is zero (0)

If $a \in R$ there exists an element $a^{/}$ then $a + a^{/} = a^{/} + a = 0$ then $a^{/}$ is called additive inverse of $a$

Or

$$a + (-a) = -a + a = 0$$

**Example:**

$$3 + (-3) = 3 - 3 = 0$$
$$-3 + 3 = 0$$

**Ex # 2.2**

**Multiplicative Inverse**

When the Product of two numbers is "1".

If $a \in R$ and $a \neq 0$ there exists an element $a^{-1} \in R$ then

$a\,.\,a^{-1} = a^{-1}\,.\,a = 1$ then $a^{-1}$ is called multiplicative inverse of $a$

**Or**

$$a\,.\,\frac{1}{a} = \frac{1}{a}.a = 1$$

**Example:**

$$3 \times \frac{1}{3} = \frac{1}{3} \times 3 = 1$$

**Distributive Property of Multiplication over Addition**

If $a,\ b,\ c\ \in R$ then

$$a(b+c) = ab + ac$$
$$(b+c)a = ba + ca$$

**Example:**

$$2(3+5) = 2 \times 3 + 2 \times 5$$
$$2(8) = 6 + 10$$
$$16 = 16$$

**Properties of Equality of Real Numbers**

**Reflexive Property of equality**

Every number is equal to itself.

$$a = a$$

**Example:**

$$3 = 3$$

**Symmetric Property of Equality**

If $a = b$ then also $b = a$

**Examples:**

$$x = 5$$
$$or\ 5 = x$$
$$x^2 = y$$
$$or\ y = x^2$$

**Transitive Property of Equality**

If $a = b$ and $b = c$ then $a = c$

**Example:**

$if\ x + y = z$ and $z = a + b$

Then $x + y = a + b$

**Ex # 2.2**

**Additive Property of Equality**

If $a = b$ then also $a + c = b + c$

**Examples:**

$$x - 3 = 5$$

Add 3 on B.S

$$x - 3 + 3 = 5 + 3$$
$$x = 8$$

---

$$x + 3 = 5$$

Subtract 3 from B.S

$$x + 3 - 3 = 5 - 3$$
$$x = 2$$

**Multiplicative Property of Equality**

If $a = b$ then also $a.\ c = b\ .\ c$

Or

$a = b$ then $\dfrac{a}{c} = \dfrac{b}{c}$

**Examples:**

$$\frac{x}{3} = 5$$

Multiply B.S by 3

$$\frac{x}{3} \times 3 = 5 \times 3$$
$$x = 15$$

---

$$2x = 24$$

Divide B.S by 2

$$\frac{2x}{2} = \frac{24}{2}$$
$$x = 12$$

**Cancellation Property w.r.t Addition**

If $a + c = b + c$ then $a = b$

**Examples:**

$$2x + 5 = y + 5$$
$$2x = y$$
$$2x - 5 = y - 5$$
$$2x = y$$

Ex # 2.2

**Cancellation Property w.r.t Multiplication**

If $a\,.\,c = b\,.\,c$ then $a = b$

**OR**

If $\frac{a}{c} = \frac{b}{c}$ then $a = b$

**Examples:**

$$2x \times 5 = y \times 5$$
$$2x = y$$
$$\frac{2x}{5} = \frac{y}{5}$$
$$2x = y$$

**Properties of Inequality of Real Numbers**

**Trichotomy Property**

Trichotomy property means when comparing two numbers, one of the following must be true:

$$a = b$$
$$a < b$$
$$a > b$$

**Examples:**

$$5 = 5$$
$$3 < 5$$
$$3 > 5$$

**Transitive Property**

(i) **If $a > b$ and $b > c$ then $a > c$**

**Example:**

If $7 > 5$ and $5 > 3$ then $\mathbf{7 > 3}$

(ii) **If $a < b$ and $b < c$ then $a < c$**

**Example:**

If $3 < 5$ and $5 < 7$ then $\mathbf{3 < 7}$

**Additive Property**

(i) **If $a < b$ then $a + c < b + c$**

**Example:**

$3 < 5$ then $3 + 2 < 5 + 2$

---

$$x - 3 > 5$$

Add 3 on B.S

$$x - 3 + 3 = 5 + 3$$
$$x = 8$$

Ex # 2.2

(ii) **If $a > b$ then $a + c > b + c$**

**Example:**

(a) $5 > 3$ then $5 - 2 > 3 - 2$

(b) $5 > 3$ then $5 - 7 > 3 - 7$ So $-2 > -4$

(c) $$x + 3 > 5$$

Subtract 3 from B.S

$$x + 3 - 3 = 5 - 3$$
$$x = 2$$

**Multiplicative Property**

**When $c > 0$:**

(i) If $a < b$ then $ac < bc$

(ii) If $a > b$ then $ac > bc$

**Example:**

(a) $5 > 3$ then $5 \times 2 > 3 \times 2$

---

(b) $$\frac{x}{3} > 5$$

Multiply B.S by 3

$$\frac{x}{3} \times 3 > 5 \times 3$$
$$x > 15$$

---

$$2x > 24$$

Divide B.S by 2

$$\frac{2x}{2} > \frac{24}{2}$$
$$x > 12$$

**When $c < 0$:**

(i) If $a < b$ then $ac > bc$

(ii) If $a > b$ then $ac < bc$

**Example:**

(a) $5 > 3$ then $5 \times -2 < 3 \times -2$ So $-10 < -6$

---

(b) $$\frac{x}{-3} < 5$$

Multiply B.S by $-3$

$$\frac{x}{-3} \times -3 > 5 \times -3$$
$$x > -15$$

**Example: 4** **Ex # 2.2**

**Page # 58**

**Solve the following equation using properties of real numbers.**

$\mathbf{2x - 5 = 3x + 4}$

**Solution:**

| | |
|---|---|
| $2x - 5 = 3x + 4$ | |
| $2x - 5 + 5 = 3x + 4 + 5$ | $\therefore$ $a = b$ *then* $a + c = b + c$ |
| $2x - 5 + 5 = 3x + 9$ | $\therefore$ *Closure Property w.r.t Additon* |
| $2x + 0 = 3x + 9$ | $\therefore$ $-5$ & $5$ *are additive inverse* |
| $2x = 3x + 9$ | $\therefore$ $0$ *is the additive identity* |
| $3x + 9 = 2x$ | $\therefore$ *Symmetric Property* |
| $3x + 9 - 2x = 2x - 2x$ | $\therefore$ $a = b$ *then* $a - c = b - c$ |
| $3x - 2x + 9 = 0$ | $\therefore$ $2x$ & $-2x$ *are additive inverse* |
| $(3 - 2)x + 9 = 0$ | $\therefore$ *Distributive Property* |
| $1.x + 9 = 0$ | |
| $x + 9 = 0$ | $\therefore$ $1$ *is Multiplicative Identity* |
| $x + 9 - 9 = 0 - 9$ | $\therefore$ $a = b$ *then* $a - c = b - c$ |
| $x + 9 - 9 = -9$ | $\therefore$ $0$ *is the Additive Identity* |
| $x + 0 = -9$ | $\therefore$ $9$ & $-9$ *are additive inverse* |
| $x = -9$ | $\therefore$ $0$ *is the Additive Identity* |

## ***Ex # 2.2***
## ***Page # 59***

**Q1: Name the properties used in following equations.**

**(i)** $\mathbf{1 + (4 + 3) = (1 + 4) + 3}$

**Ans:** Associative law of addition

**(ii)** $\mathbf{5(a + b) = 5a + 5b}$

**Ans:** Distributive law of multiplication over addition

**(iii)** $\mathbf{a + 0 = 0 + a = a}$

**Ans:** Additive identity

**(iv)** $\mathbf{5 \times \frac{1}{5} = \frac{1}{5} \times 5 = 1}$

**Ans:** Multiplicative inverse

**Q2: Write the missing number.**

**(i)** $\mathbf{2 + (\_\_\_ + 4) = (2 + 6) + 4}$

Answer: 6

**(ii)** $\mathbf{7 + (4 + 2) = 13}$, **so** $\mathbf{(7 + 4) + 2 =}$____

Answer: 13

**(iii)** $\mathbf{9 \times (3 \times 4) = 108}$, **so** $\mathbf{(9 \times 3) \times 4 =}$_____

Answer: 108

**(iv)** $\mathbf{5 \times (8 \times 9) = (5 \times \_\_\_\_) \times 9}$

Answer: 8

**Q3: Chose the correct option**

**(i)** $\mathbf{8 \times (6 \times 7)}$ **is equal to:**

**(a)** $8 \times 6 - 7$ **(b)** $8 - (6 - 7)$

**(c)** $8 \times 12$ **(d)** $(8 \times 6) \times 7$

Answer: d. $(8 \times 6) \times 7$

**(ii) Which one of the following illustrates the Associative Law of Addition?**

**(a)** $3 + (2 + 4) = (4 + 4) + 1$

**(b)** $3 + (2 + 4) = (3 + 2) + 4$

**(c)** $3 + (2 + 4) = (5 + 2) + 2$

**(d)** $3 + (2 + 4) = (2 + 6) + 1$

**Answer: b**. $3 + (2 + 4) = (3 + 2) + 4$

**Ex # 2.2**

**(iii)** Which one of the following illustrates the Associative Law of Multiplication?

**(a)** $4 \times (3 \times 6) = (6 \times 6) \times 2$

**(b)** $4 \times (3 \times 6) = (3 \times 12) \times 2$

**(c)** $4 \times (3 \times 6) = (4 \times 3) \times 6$

**(d)** $4 \times (3 \times 6) = (3 \times 8) \times 3$

**Answer: c**. $4 \times (3 \times 6) = (4 \times 3) \times 6$

**Q4: Do this without using distributive property.**

**(i)** $\mathbf{39 \times 63 + 39 \times 37}$

**Solution:**

$39 \times 63 + 39 \times 37$
$= 2457 + 1443$
$= 3900$

**(ii)** $\mathbf{81 \times 450 + 81 \times 550}$

**Solution:**

$81 \times 450 + 81 \times 550$
$= 36450 + 44550$
$= 81000$

**(iii)** $\mathbf{50 \times 161 - 50 \times 81}$

**Solution:**

$50 \times 161 - 50 \times 81$
$= 8050 - 4050$
$= 4000$

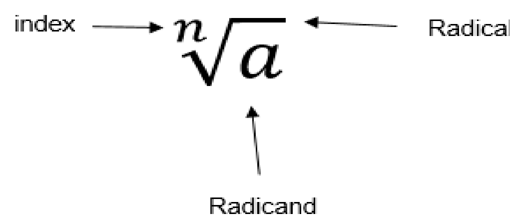
**(iv)** $\mathbf{827 \times 60 - 327 \times 60}$

**Solution:**

$827 \times 60 - 327 \times 60$
$= 49620 - 19620$
$= 30000$

**Ex # 2.3**

**RADICALS AND RADICANDS**




$\sqrt[n]{a}$ is the radical form of the nth root of $a$.

$a^{\frac{1}{n}}$ is the exponential form of the nth root of $a$.

If $n = 2$ then it becomes square root and write $\sqrt{a}$ instead of $\sqrt[2]{a}$

If $n = 3$ then it is called cube root like $\sqrt[3]{a}$

If $n = 5$ then it is called 5th root like $\sqrt[5]{625}$

**Important Notes**

**(i)** If $a$ is positive, then the $nth$ root of $a$ is also positive.

**Example:**

$$\sqrt[3]{64} = \sqrt[3]{(4)^3} = 4$$

**(ii)** If $a$ is negative, then $n$ must be odd for the $nth$ root of $a$ to be a real number.

**Example:**

$$\sqrt[3]{\mathbf{-64}} = \sqrt[3]{(\mathbf{-4})^{\mathbf{3}}} = \mathbf{-4}$$

**(iii)** If $a$ is zero, then $\sqrt[n]{0} = 0$

**Properties of Radicals:**

**Product Rule of Radicals:**

$\sqrt[n]{ab} = \sqrt[n]{a} \,.\, \sqrt[n]{b}$

**Example:**

$\sqrt{6x}\sqrt{6y^2}$
$\sqrt{(6x)(6y^2)} = \sqrt{36y^2x} = \sqrt{36}\sqrt{y^2}\sqrt{x}$
$= 6y\sqrt{x}$

$\sqrt{6x}\sqrt{6x^2}$
$\sqrt{(6x)(6x^2)} = \sqrt{36x^2x} = \sqrt{36}\sqrt{x^2}\sqrt{x}$
$= 6y\sqrt{x}$

**Ex # 2.3**

**Quotient Rule of Radicals:**

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

**Example:**

Simplify: $2\sqrt{\frac{150xy}{3x}}$

**Solution:**

$$2\sqrt{\frac{150xy}{3x}} = 2\sqrt{50y} = 2\sqrt{5\times5\times2y}$$

$$= 2\sqrt{5^2}\sqrt{2y} = 2(5)\sqrt{2y} = 10\sqrt{2y}$$

**Radical form of an Expression:**

The number or quantity that is written under a radical sign ( $\sqrt{\ }$ or $\sqrt[n]{\ }$ ) is called radical form of an expression.

**Example:**

$\sqrt{9}$ is the radical form of 3.

**Exponential form of an Expression:**

The number or quantity that is written in the form of exponent is called exponential form of an expression.

**Example:**

$3^2$ is the exponential form of 9.

| **Radical Form** | **Exponential Form** |
|---|---|
| $\sqrt[n]{a}$ | $a^{\frac{1}{n}}$ |
| $\sqrt[n]{a^m}$ or $(\sqrt[n]{a})^m$ | $a^{\frac{m}{n}}$ |
| $\sqrt[n]{a^n}$ | $a$ |



**Some frequently used radicals are given in the following table**

| **Square Root** | **Cube Root** | **Fourth Root** |
|---|---|---|
| $\sqrt{1} = 1$ | $\sqrt[3]{1} = 1$ | $\sqrt[4]{1} = 1$ |
| $\sqrt{4} = 2$ | $\sqrt[3]{8} = 2$ | $\sqrt[4]{16} = 2$ |
| $\sqrt{9} = 3$ | $\sqrt[3]{27} = 3$ | $\sqrt[4]{81} = 3$ |
| $\sqrt{16} = 4$ | $\sqrt[3]{64} = 4$ | $\sqrt[4]{256} = 4$ |
| $\sqrt{25} = 5$ | $\sqrt[3]{125} = 5$ | $\sqrt[4]{625} = 5$ |
| $\sqrt{36} = 6$ | $\sqrt[3]{216} = 6$ | $\sqrt[4]{1296} = 6$ |

**Example 5 Page # 61**

What is the difference between (i) $x^2 = 16$ (ii) $x = \sqrt{16}$ ?

(i) $x^2 = 16$

**Solution:**

$$x^2 = 16$$

This means what numbers squared becomes 16. Thus $x$ can be 4 or $-4$ like $(4)^2 = 16$ and also $(-4)^2 = 16$.

Hence the value of $x = \pm 4$.

(ii) $x = \sqrt{16}$

**Solution:**

$$x = \sqrt{16}$$

Here $x$ is the principal square root of 16, which has always a positive value such is $x = 4$.

# Ex # 2.3

**Page # 64**

**Q1: 64** **Write down the index and radicand for each of the following expressions.**

**(i)** $\sqrt{\dfrac{11}{y}}$

$index = 2, radicand = \dfrac{11}{y}$

**(ii)** $\sqrt[3]{\dfrac{13}{3x}}$

$index = 3, radicand = \dfrac{13}{3x}$

**(iii)** $\sqrt[5]{ab^2}$

$index = 5, radicand = ab^2$

**Q2: 64** **Transform the following radical forms into exponential forms. Do not simplify.**

**(i)** $\sqrt{36}$

Exponential form= $(36)^{\frac{1}{2}}$

**(ii)** $\sqrt{1000}$

Exponential form= $(1000)^{\frac{1}{2}}$

**(iii)** $\sqrt[3]{8}$

Exponential form= $(8)^{\frac{1}{3}}$

**(iv)** $\sqrt[n]{q}$

Exponential form= $(q)^{\frac{1}{n}}$

**(v)** $\sqrt{(5-6a^2)^3}$

$((5-6a^2)^3)^{\frac{1}{2}}$

Exponential form= $(5-6a^2)^{\frac{3}{2}}$

**(vi)** $\sqrt[3]{-64}$

Exponential form= $(-64)^{\frac{1}{3}}$

**Ex # 2.3**

**Q3: 64** **Transform the following exponential form of an expression into radical form.**

**(i)** $-7^{\frac{1}{3}}$

$-\sqrt[3]{7}$

**(ii)** $x^{-\frac{3}{2}}$

$(x^{-3})^{\frac{1}{2}}$

$\sqrt{x^{-3}}$

**(iii)** $(-8)^{\frac{1}{5}}$

$\sqrt[5]{-8}$

**(iv)** $y^{\frac{3}{4}}$

$(y^3)^{\frac{1}{4}}$

$\sqrt[4]{y^3}$

**(v)** $b^{\frac{4}{5}}$

$(b^4)^{\frac{1}{5}}$

$\sqrt[5]{b^4}$

**(vi)** $(3x)^{\frac{1}{q}}$

$\sqrt[q]{3x}$



**Q4:** **Simplify:**

**(i)** $\sqrt[3]{125x}$

**Solution:**

$\sqrt[3]{125x}$

$= (125x)^{\frac{1}{3}}$

$= (125)^{\frac{1}{3}}(x)^{\frac{1}{3}}$

$= (5 \times 5 \times 5)^{\frac{1}{3}}(x)^{\frac{1}{3}}$

$= (5^3)^{\frac{1}{3}}(x)^{\frac{1}{3}}$

$= 5(x)^{\frac{1}{3}}$

$= 5\sqrt[3]{x}$

**Ex # 2.3**

**(ii)** $\sqrt[3]{\dfrac{8}{27}}$

$= \left(\dfrac{8}{27}\right)^{\frac{1}{3}}$

$= \left(\dfrac{2 \times 2 \times 2}{3 \times 3 \times 3}\right)^{\frac{1}{3}}$

$= \left(\dfrac{2^3}{3^3}\right)^{\frac{1}{3}}$

$= \dfrac{(2^3)^{\frac{1}{3}}}{(3^3)^{\frac{1}{3}}}$

$= \dfrac{2}{3}$

**(iii)** $\sqrt{\dfrac{\mathbf{625x^3y^4}}{\mathbf{25xy^2}}}$

**Solution:**

$\sqrt{\dfrac{625x^3y^4}{25xy^2}}$

$= \sqrt{25x^2y^2}$

$= (25x^2y^2)^{\frac{1}{2}}$

$= (25)^{\frac{1}{2}}(x^2)^{\frac{1}{2}}(y^2)^{\frac{1}{2}}$

$= 5xy$

**(iv)** $\sqrt{(\mathbf{3y-5})^2}$

**Solution:**

$\sqrt{(3y-5)^2}$

$= [(3y-5)^2]^{\frac{1}{2}}$

$= 3y - 5$

**Ex # 2.3**

**(v)** $\mathbf{6\sqrt{18}}$

**Solution:**

$6\sqrt{18}$

$= 6(18)^{\frac{1}{2}}$

$= 6(3 \times 3 \times 2)^{\frac{1}{2}}$

$= 6(3^2 \times 2)^{\frac{1}{2}}$

$= 6(3^2)^{\frac{1}{2}}(2)^{\frac{1}{2}}$

$= 6(3)\sqrt{2}$

$= 18\sqrt{2}$

**(vi)** $\sqrt[3]{\mathbf{54x^3y^3z^2}}$

**Solution:**

$\sqrt[3]{54x^3y^3z^2}$

$= (54x^3y^3z^2)^{\frac{1}{3}}$

$= (54)^{\frac{1}{3}}(x^3)^{\frac{1}{3}}(y^3)^{\frac{1}{3}}(z^2)^{\frac{1}{3}}$

$= (3 \times 3 \times 3 \times 2)^{\frac{1}{3}}(x)(y)(z^2)^{\frac{1}{3}}$

$= (3^3 \times 2)^{\frac{1}{3}}(x)(y)(z^2)^{\frac{1}{3}}$

$= (3^3)^{\frac{1}{3}}(2)^{\frac{1}{3}}(x)(y)(z^2)^{\frac{1}{3}}$

$= (3)(x)(y)(2)^{\frac{1}{3}}(z^2)^{\frac{1}{3}}$

$= 3xy(2z^2)^{\frac{1}{3}}$

$= 3xy\sqrt[3]{2z^2}$

# Ex # 2.4

## Base

جس کے اوپر power ہوا سے Base کہتے ہیں۔

## Exponent /Power

Base کے اوپر جو چھوٹا سا نمبر ہوتا ہے اسے power کہتے ہیں۔ اس کو index بھی کہتے ہیں۔

## Co-efficient

Base کے Left طرف جو نمبر ہوتا ہے اسے Co-efficient کہتے ہیں۔

Base اور Co-efficient آپس میں Multiply ہوتے ہیں

| $4x^2$ | $5y^{-3}$ | $-2y^3$ |
|---|---|---|
| Base: $x$<br>Power: 2<br>Co-efficient: 4 | Base: $y$<br>Power: $-3$<br>Co-efficient: 5 | Base: $y$<br>Power: 3<br>Co-efficient: $-2$ |
| $x$ | $x^3$ | $5z$ |
| Base: $x$<br>Power: 1<br>Co-efficient: 1 | Base: $x$<br>Power: 3<br>Co-efficient: 1 | Base: z<br>Power: 1<br>Co-efficient: 5 |

Note:

$2^{-4} = \frac{1}{2^4} = \frac{1}{16}$ $\quad$ $\frac{1}{3^{-3}} = 3^3 = 27$

$-4x^{-2} = \frac{-4}{x^2}$ $\quad$ $(a+b)^{-1} = \frac{1}{(a+b)}$

## Laws of Exponents

### Multiplication of Same Bases

To multiply powers of the same base, keep the same base and add the exponents.

**اگر ایک جیسے bases آپس میں multiply ہوتے ہیں تو:**

- ❖ **Co-efficient کو multiply کریں گے**
- ❖ **Base ایک لکھیں گے**
- ❖ **Powers کو Add کریں گے**

**Example:**

$$a^m.a^n = a^{m+n}$$

Ex # 2.4

### Multiplication of Different Bases

When different bases are multiplied just multiply the co-efficient or constant.

**اگر مختلف bases آپس میں multiply ہوتے ہیں تو صرف Co-efficient کو multiply کریں گے**

### Law of Quotient

To divide two expressions with the same bases and different exponents, keep the same base and subtract the exponents.

جب fraction میں ایک جیسے bases ہو تو اس base کو اوپر لے جائیں گے لیکن اس کے power کا sign تبدیل ہو جائے گا۔

- ❖ اگر plus ہو گا تو minus ہو جائے گا
- ❖ اگر minus ہو گا تو plus ہو جائے گا

### Law of Power of Power

To raise an exponential expression to a power, keep the same base multiply the exponents.

**جب کسی بریکٹ کے اوپر Power آجائیں تو اس کو تمام Bases کے Powers کے ساتھ Multiply کریں گے۔**

**اگر Base یا Co-efficient کے ساتھ minus کا sign ہو تو:**

**1) جب power میں even نمبر ہو تو expression کے ساتھ plus کا sign لگائیں گے۔**

$(-x)^{22} = x^{22}$ $\quad$ $(-4y)^2 = 16y^2$

**2) جب power میں Odd نمبر ہو تو expression کے ساتھ plus کا minus لگائیں گے۔**

$(-x)^{25} = -x^{25}$ $\quad$ $(-2y)^3 = -8y^3$

### Zero Exponent Rule

Any non-zero number raised to the zero power equals one.

کسی بھی Base کا Power اگر Zero ہو تو 1 کے برابر ہو گا۔

$100^0 = 1$ and $(xy)^0 = 1$

## Ex # 2.4

**Page # 67**

**Q1:** **Write the base, exponent and value of the following.**

**(i)** $(2)^{-9} = \frac{1}{1024}$

$base = 2,\ Exponent = -9, value = \frac{1}{1024}$

**(ii)** $\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$

$base = \frac{a}{b},\ Exponent = p,\ \ value = \frac{a^p}{b^p}$

**(iii)** $(-4)^2 = 16$

$base = -4,\ \ Exponent = 2,\ \ value = 16$

**Q2: 67** **If a, b denote the real numbers then simplify the following.**

**(i)** $a^3 \times a^5$

**Solution:**

$a^3 \times a^5$

$= a^{3+5}$

$= a^8$

**(ii)** $\left(\frac{b}{a}\right)^{\frac{3}{2}} \left(\frac{b}{a}\right)^{-\frac{2}{3}}$

**Solution:**

$\left(\frac{b}{a}\right)^{\frac{3}{2}} \left(\frac{b}{a}\right)^{-\frac{2}{3}}$

$= \left(\frac{b}{a}\right)^{\frac{3}{2}-\frac{2}{3}}$

$= \left(\frac{b}{a}\right)^{\frac{9-4}{6}}$

$= \left(\frac{b}{a}\right)^{\frac{5}{6}}$

**(iii)** $(-a)^4 \times (-a)^3$

**Solution:**

$(-a)^4 \times (-a)^3$

$= (-a)^{4+3}$

$= (-a)^7$

$= -a^7$

Ex # 2.4

**(iv)** $(-2a^2b^3)^3$

**Solution:**

$(-2a^2b^3)^3$

$= (-2)^3 a^{2\times3} b^{3\times3}$

$= -8a^6b^9$

**(v)** $a^3(-2b)^2$

**Solution:**

$= a^3(-2b)^2$

$= a^3(-2)^2(b)^2$

$= a^3 \times 4b^2$

$= 4a^3b^2$

**(vi)** $(a^2b)(a^2b)$

**Solution:**

$(a^2b)(a^2b)$

$= a^{2+2}b^{1+1}$

$= a^4b^2$

**(vii)** $\frac{a^0 . b^0}{2}$

**Solution:**

$\frac{a^0 . b^0}{2}$

$= \frac{1 \times 1}{2}$

$= \frac{1}{2}$

**(viii)** $(-3a^2b^2)^2$

**Solution:**

$(-3a^2b^2)^2$

$= (-3)^2 a^{2\times2} b^{2\times2}$

$= 9a^4b^4$

**Ex # 2.4**

**(ix)** $\left(\frac{a^2}{b^4}\right)^{\frac{3}{2}}$

**Solution:**

$$\left(\frac{a^2}{b^4}\right)^{\frac{3}{2}}$$
$$= \frac{a^{2\times\frac{3}{2}}}{b^{4\times\frac{3}{2}}}$$
$$= \frac{a^{1\times3}}{b^{2\times3}}$$
$$= \frac{a^3}{b^6}$$

**Q3: Simplify the following.**

**(i)** $\frac{7^6}{7^4}$

**Solution:**

$$\frac{7^6}{7^4}$$
$$= 7^6.7^{-4}$$
$$= 7^{6-4}$$
$$= 7^2$$

**(ii)** $\frac{2^4.5^3}{10^2}$

**Solution:**

$$\frac{2^4.5^3}{10^2}$$
$$= \frac{2^4.5^3}{(2\times5)^2}$$
$$= \frac{2^4.5^3}{2^2.5^2}$$
$$= 2^4.5^3.2^{-2}.5^{-2}$$
$$= 2^{4-2}.5^{3-2}$$
$$= 2^2.5^1$$
$$= 4\times5$$
$$= 20$$

**Ex # 2.4**

**(iii)** $\left\{\frac{(a+b)^2.(c+d)^3}{(a+b).(c+d)^2}\right\}^3$

**Solution:**

$$\left\{\frac{(a+b)^2.(c+d)^3}{(a+b).(c+d)^2}\right\}^3$$
$$= \frac{(a+b)^{2\times3}.(c+d)^{3\times3}}{(a+b)^{1\times3}.(c+d)^{2\times3}}$$
$$= \frac{(a+b)^6.(c+d)^9}{(a+b)^3.(c+d)^6}$$
$$= (a+b)^6.(c+d)^9.(a+b)^{-3}.(c+d)^{-6}$$
$$= (a+b)^{6-3}.(c+d)^{9-6}$$
$$= (a+b)^3.(c+d)^3$$

**(iv)** $(\sqrt[3]{a})^{\frac{1}{2}}$

**Solution:**

$$(\sqrt[3]{a})^{\frac{1}{2}}$$
$$= \left(a^{\frac{1}{3}}\right)^{\frac{1}{2}}$$
$$= a^{\frac{1}{3}\times\frac{1}{2}}$$
$$= a^{\frac{1}{6}}$$

**(v)** $\sqrt[5]{x^5}.\sqrt[4]{x^4}$

**Solution:**

$$\sqrt[5]{x^5}.\sqrt[4]{x^4}$$
$$= (x^5)^{\frac{1}{5}}(x^4)^{\frac{1}{4}}$$
$$= (x)^{5\times\frac{1}{5}}.(x)^{4\times\frac{1}{4}}$$
$$= x.x$$
$$= x^2$$

**Ex # 2.4**

**Q4: 67** **Simplify the following in such a way that no answers should contain fractional or negative exponent.**

**(i)** $\left(\frac{25}{81}\right)^{\frac{1}{2}}$

**Solution:**

$$\left(\frac{25}{81}\right)^{\frac{1}{2}}$$
$$=\left(\frac{5\times5}{9\times9}\right)^{\frac{1}{2}}$$
$$=\left(\frac{5^2}{9^2}\right)^{\frac{1}{2}}$$
$$=\frac{5^{2\times\frac{1}{2}}}{9^{2\times\frac{1}{2}}}$$
$$=\frac{5}{9}$$

**(ii)** $\dfrac{(ab)^{\frac{1}{b}}}{\left(\frac{1}{ab}\right)^{\frac{1}{a}}}$

**Solution:**

$$\frac{(ab)^{\frac{1}{b}}}{\left(\frac{1}{ab}\right)^{\frac{1}{a}}}$$
$$=\frac{(ab)^{\frac{1}{b}}}{((ab)^{-1})^{\frac{1}{a}}}$$
$$=\frac{(ab)^{\frac{1}{b}}}{(ab)^{-\frac{1}{a}}}$$
$$=(ab)^{\frac{1}{b}}.(ab)^{\frac{1}{a}}$$
$$=(ab)^{\frac{1}{b}+\frac{1}{a}}$$
$$=(ab)^{\frac{a+b}{ba}}$$
$$=(ab)^{\frac{a+b}{ab}}$$
$$=a^{\frac{a+b}{ab}}.b^{\frac{a+b}{ab}}$$

**Ex # 2.4**

**(iii)** $\dfrac{2^{p+1}.3^{2p-q}.5^{p+q}.6^q}{6^p.10^{q+2}.15^p}$

**Solution:**

$$\frac{2^{p+1}.3^{2p-q}.5^{p+q}.6^q}{6^p.10^{q+2}.15^p}$$
$$=\frac{2^{p+1}.3^{2p-q}.5^{p+q}.(2\times3)^q}{(2\times3)^p.(2\times5)^{q+2}.(3\times5)^p}$$
$$=\frac{2^{p+1}.3^{2p-q}.5^{p+q}.2^q.3^q}{2^p.3^p.2^{q+2}.5^{q+2}.3^p.5^p}$$
$$=\frac{2^{p+1+q}.3^{2p-q+q}.5^{p+q}}{2^{p+q+2}.3^{p+p}.5^{q+2+p}}$$
$$=\frac{2^{p+1+q}.3^{2p}.5^{p+q}}{2^{p+q+2}.3^{2p}.5^{q+2+p}}$$
$$=2^{p+1+q}.3^{2p}.5^{p+q}.2^{-p-q-2}.3^{-2p}.5^{-q-2-p}$$
$$=2^{p+1+q-p-q-2}.3^{2p-2p}.5^{p+q-q-2-p}$$
$$=2^{1-2}.3^0.5^{-2}$$
$$=2^{-1}.3^0.5^{-2}$$
$$=\frac{1}{2}\times1\times\frac{1}{5^2}$$
$$=\frac{1}{2}\times1\times\frac{1}{25}$$
$$=\frac{1}{50}$$

**(iv)** $\left(\dfrac{x^p}{x^q}\right)^{p+q}\left(\dfrac{x^q}{x^r}\right)^{q+r}\left(\dfrac{x^r}{x^p}\right)^{r+p}$

**Solution:**

$$\left(\frac{x^p}{x^q}\right)^{p+q}\left(\frac{x^q}{x^r}\right)^{q+r}\left(\frac{x^r}{x^p}\right)^{r+p}$$
$$=(x^p.x^{-q})^{p+q}(x^q.x^{-r})^{q+r}(x^r.x^{-p})^{r+p}$$
$$=(x^{p-q})^{p+q}(x^{q-r})^{q+r}(x^{r-p})^{r+p}$$
$$=(x)^{(p-q)(p+q)}.(x)^{(q-r)(q+r)}.(x)^{(r-p)(r+p)}$$
$$=(x)^{p^2-q^2}.(x)^{q^2-r^2}.(x)^{r^2-p^2}$$
$$=x^{p^2-q^2+q^2-r^2+r^2-p^2}$$
$$=x^0$$
$$=1$$

**Ex # 2.4**

**Q5: 67** **Prove that** $\left(\frac{4^5.64^3.2^3}{8^5.(128)^2}\right)^{\frac{1}{2}} = 2$

**Solution:**

$$\left(\frac{4^5.64^3.2^3}{8^5.(128)^2}\right)^{\frac{1}{2}} = 2$$

L.H.S

$$= \left(\frac{(2^2)^5.(2^6)^3.2^3}{(2^3)^5.(2^7)^2}\right)^{\frac{1}{2}}$$

$$= \left(\frac{2^{10}.2^{18}.2^3}{2^{15}.2^{14}}\right)^{\frac{1}{2}}$$

$$= \left(\frac{2^{10+18+3}}{2^{15+14}}\right)^{\frac{1}{2}}$$

$$= \left(\frac{2^{31}}{2^{29}}\right)^{\frac{1}{2}}$$

$$= (2^{31-29})^{\frac{1}{2}}$$

$$= (2^2)^{\frac{1}{2}}$$

$$= 2^{2\times\frac{1}{2}}$$

$$= 2$$

=R.H.S

**Ex # 2.5**

**Complex Number**

A number of the form $a + bi$ where $a$ and $b$ are real numbers is called complex number where "$a$" is called real part and "b" is called imaginary part.

**Conjugate of a Complex Numbers**

A conjugate of a complex number is obtained by changing the sign of imaginary part. The conjugate of $a + bi$ is $a - bi$ or the conjugate of $a + bi$ is denoted by $\overline{a + bi} = a - bi$.

**Ex # 2.5**

**Equality of Two Complex Numbers**

Let $Z_1 = a + bi$ and $Z_2 = c + di$ then $Z_1 = Z_2$ if real parts are equal i.e. $a = c$ and imaginary parts are equal i.e. $b = d$.

**Operation on Complex Numbers**

**Addition of Complex Numbers**

Let $Z_1 = a + bi$ and $Z_2 = c + di$ then

$Z_1 + Z_2 = (a + bi) + (c + di)$

$Z_1 + Z_2 = a + bi + c + di$

$Z_1 + Z_2 = a + c + bi + di$

$Z_1 + Z_2 = (a + c) + (b + d)i$

**Subtraction of Complex Numbers**

Let $Z_1 = a + bi$ and $Z_2 = c + di$ then

$Z_1 - Z_2 = (a + bi) - (c + di)$

$Z_1 - Z_2 = a + bi - c - di$

$Z_1 - Z_2 = a - c + bi - di$

$Z_1 - Z_2 = (a - c) + (b - d)i$

**Multiplication of Complex Numbers**

Let $Z_1 = a + bi$ and $Z_2 = c + di$ then

$Z_1.Z_2 = (a + bi)(c + di)$

$Z_1.Z_2 = ac + adi + bci + bdi^2$

$Z_1.Z_2 = ac + (ad + bc)i + bd(-1)$ as $i^2 = -1$

$Z_1.Z_2 = ac + (ad + bc)i - bd$

$Z_1.Z_2 = (ac - bd) + (ad + bc)i$

**Division of Complex Numbers**

Let $Z_1 = a + bi$ and $Z_2 = c + di$ then

$$\frac{Z_1}{Z_2} = \frac{a + bi}{c + di}$$

Multiply and Divide by $c - di$

$$\frac{Z_1}{Z_2} = \frac{a + bi}{c + di} \times \frac{c - di}{c - di}$$

$$\frac{Z_1}{Z_2} = \frac{(a + bi)(c - di)}{(c + di)(c - di)}$$

$$\frac{Z_1}{Z_2} = \frac{ac - adi + bci - bdi^2}{c^2 - (di)^2}$$

**Ex # 2.5**

$$\frac{Z_1}{Z_2}=\frac{ac+bci-adi-bd(-1)}{c^2-d^2i^2} \quad As\ i^2=-1$$

$$\frac{Z_1}{Z_2}=\frac{ac+(bc-ad)i+bd}{c^2-d^2(-1)}$$

$$\frac{Z_1}{Z_2}=\frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$$

$$\frac{Z_1}{Z_2}=\frac{(ac+bd)}{c^2+d^2}+\frac{(bc-ad)i}{c^2+d^2}$$

# Ex # 2.5

**Page # 71**

**Q1: Add the following complex number**

**(i) $8+9i, 5+2i$**

**Solution:**

$8+9i, 5+2i$

Let $Z_1=8+9i$

And $Z_2=5+2i$

Now

$Z_1+Z_2=(8+9i)+(5+2i)$

$Z_1+Z_2=8+9i+5+2i$

$Z_1+Z_2=8+5+9i+2i$

$Z_1+Z_2=13+11i$

**(ii) $6+3i, 3-5i$**

**Solution:**

$6+3i, 3-5i$

Let $Z_1=6+3i$

And $Z_2=3-5i$

Now

$Z_1+Z_2=(6+3i)+(3-5i)$

$Z_1+Z_2=6+3i+3-5i$

$Z_1+Z_2=6+3+3i-5i$

$Z_1+Z_2=9-2i$

**(iii) $2i+3, 8-5\sqrt{-1}$**

**Solution:**

$2i+3, 8-5\sqrt{-1}$

Let $Z_1=2i+3$

And $Z_2=8-5\sqrt{-1}$

$8-5i \quad \because \sqrt{-1}=i$

**Ex # 2.5**

Now

$Z_1+Z_2=(2i+3)+(8-5i)$

$Z_1+Z_2=2i+3+8-5i$

$Z_1+Z_2=3+8+2i-5i$

$Z_1+Z_2=11-3i$

**(iv) $\sqrt{3}+\sqrt{2}i, 3\sqrt{3}-2\sqrt{2}i$**

**Solution:**

$\sqrt{3}+\sqrt{2}i, 3\sqrt{3}-2\sqrt{2}i$

Let $Z_1=\sqrt{3}+\sqrt{2}i$

And $Z_2=3\sqrt{3}-2\sqrt{2}i$

Now

$Z_1+Z_2=\left(\sqrt{3}+\sqrt{2}i\right)+\left(3\sqrt{3}-2\sqrt{2}i\right)$

$Z_1+Z_2=\sqrt{3}+\sqrt{2}i+3\sqrt{3}-2\sqrt{2}i$

$Z_1+Z_2=\sqrt{3}+3\sqrt{3}+\sqrt{2}i-2\sqrt{2}i$

$Z_1+Z_2=4\sqrt{3}-\sqrt{2}i$

**Q2: Subtract:**

**(i) $-2+3i$ from $6-3i$**

**Solution:**

$-2+3i$ from $6-3i$

Let $Z_1=-2+3i$

And $Z_2=6-3i$

Now

$Z_2-Z_1=(6-3i)-(-2+3i)$

$Z_2-Z_1=6-3i+2-3i$

$Z_2-Z_1=6+2-3i-3i$

$Z_2-Z_1=8-6i$

**(ii) $9+4i$ from $9-8i$**

**Solution:**

$9+4i$ from $9-8i$

Let $Z_1=9+4i$

And $Z_2=9-8i$

Now

$Z_2-Z_1=(9-8i)-(9+4i)$

$Z_2-Z_1=9-8i-9-4i$

$Z_2-Z_1=9-9-8i-4i$

$Z_2-Z_1=0-12i$

$Z_2-Z_1=-12i$

**Ex # 2.5**

**(iii)** $\mathbf{1-3i}$ **from** $\mathbf{8-i}$

**Solution:**

$1-3i$ from $8-i$

Let $Z_1 = 1-3i$

And $Z_2 = 8-i$

Now

$Z_2 - Z_1 = (8-i)-(1-3i)$

$Z_2 - Z_1 = 8-i-1+3i$

$Z_2 - Z_1 = 8-1-i+3i$

$Z_2 - Z_1 = 7+2i$

**(iv)** $6-7i$ from $6+7i$

**Solution:**

$6-7i$ from $6+7i$

Let $Z_1 = 6-7i$

And $Z_2 = 6+7i$

Now

$Z_2 - Z_1 = (6+7i)-(6-7i)$

$Z_2 - Z_1 = 6+7i-6+7i$

$Z_2 - Z_1 = 6-6+7i+7i$

$Z_2 - Z_1 = 0+14i$

$Z_2 - Z_1 = 14i$

**Q3: Multiply the following complex numbers**

**(i)** $\mathbf{1+2i, 3-8i}$

**Solution:**

$1+2i, 3-8i$

Let $Z_1 = 1+2i$

And $Z_2 = 3-8i$

Now

$Z_1.Z_2 = (1+2i)(3-8i)$

$Z_1.Z_2 = 1(3-8i)+2i(3-8i)$

$Z_1.Z_2 = 3-8i+6i-16i^2$

$Z_1.Z_2 = 3-2i-16(-1)$

$Z_1.Z_2 = 3-2i+16$

$Z_1.Z_2 = 3+16-2i$

$Z_1.Z_2 = 19-2i$

**(ii)** $\mathbf{2i, 4-7i}$

**Solution:**

$2i, 4-7i$

Let $Z_1 = 2i$

And $Z_2 = 4-7i$

**Ex # 2.5**

Now

$Z_1.Z_2 = (2i)(4-7i)$

$Z_1.Z_2 = 2i(4-7i)$

$Z_1.Z_2 = 8i-14i^2$

$Z_1.Z_2 = 8i-14(-1)$

$Z_1.Z_2 = 8i+14$

$Z_1.Z_2 = 14+8i$

**(iii)** $\mathbf{5-3i, 2-4i}$

**Solution:**

$5-3i, 2-4i$

Let $Z_1 = 5-3i$

And $Z_2 = 2-4i$

Now

$Z_1.Z_2 = (5-3i)(2-4i)$

$Z_1.Z_2 = 5(2-4i)-3i(2-4i)$

$Z_1.Z_2 = 10-20i-6i+12i^2$

$Z_1.Z_2 = 10-26i+12(-1)$

$Z_1.Z_2 = 10-26i-12$

$Z_1.Z_2 = 10-12-26i$

$Z_1.Z_2 = -2-26i$

**(iv)** $\mathbf{\sqrt{2}+i, 1-\sqrt{2}i}$

**Solution:**

$\sqrt{2}+i, 1-\sqrt{2}i$

Let $Z_1 = \sqrt{2}+i$

And $Z_2 = 1-\sqrt{2}i$

Now

$Z_1.Z_2 = (\sqrt{2}+i)(1-\sqrt{2}i)$

$Z_1.Z_2 = \sqrt{2}(1-\sqrt{2}i)+i(1-\sqrt{2}i)$

$Z_1.Z_2 = \sqrt{2}-\sqrt{2\times 2}i+1i-\sqrt{2}i^2$

$Z_1.Z_2 = \sqrt{2}-2i+1i-\sqrt{2}(-1)$

$Z_1.Z_2 = \sqrt{2}-i+\sqrt{2}$

$Z_1.Z_2 = \sqrt{2}+\sqrt{2}-i$

$Z_1.Z_2 = 2\sqrt{2}-i$

**Ex # 2.5**

**Q4: Divide the first complex number by the second.**

**(i)** $\boldsymbol{Z_1 = 2 + i, Z_2 = 5 - i}$

**Solution:**

$Z_1 = 2 + i, Z_2 = 5 - i$

$$\frac{Z_1}{Z_2} = \frac{2+i}{5-i}$$

Multiply and divide by $5 + i$

$$\frac{Z_1}{Z_2} = \frac{2+i}{5-i} \times \frac{5+i}{5+i}$$

$$\frac{Z_1}{Z_2} = \frac{(2+i)(5+i)}{(5-i)(5+i)}$$

$$\frac{Z_1}{Z_2} = \frac{10+2i+5i+i^2}{(5)^2-(i)^2}$$

$$\frac{Z_1}{Z_2} = \frac{10+7i+(-1)}{25-i^2}$$

$$\frac{Z_1}{Z_2} = \frac{10+7i-1}{25-(-1)}$$

$$\frac{Z_1}{Z_2} = \frac{10-1+7i}{25+1}$$

$$\frac{Z_1}{Z_2} = \frac{9+7i}{26}$$

$$\frac{Z_1}{Z_2} = \frac{9}{26} + \frac{7}{26}i$$

**(ii)** $\boldsymbol{Z_1 = 3i + 4, Z_2 = 1 - i}$

**Solution:**

$Z_1 = 3i + 4$

$4 + 3i$

$Z_2 = 1 - i$

$$\frac{Z_1}{Z_2} = \frac{4+3i}{1-i}$$

Multiply and divide by $1 + i$

$$\frac{Z_1}{Z_2} = \frac{4+3i}{1-i} \times \frac{1+i}{1+i}$$

**Ex # 2.5**

$$\frac{Z_1}{Z_2} = \frac{(4+3i)(1+i)}{(1-i)(1+i)}$$

$$\frac{Z_1}{Z_2} = \frac{4+4i+3i+3i^2}{(1)^2-(i)^2}$$

$$\frac{Z_1}{Z_2} = \frac{4+7i+3(-1)}{1-i^2}$$

$$\frac{Z_1}{Z_2} = \frac{4+7i-3}{1-(-1)}$$

$$\frac{Z_1}{Z_2} = \frac{4-3+7i}{1+1}$$

$$\frac{Z_1}{Z_2} = \frac{1+7i}{2}$$

$$\frac{Z_1}{Z_2} = \frac{1}{2} + \frac{7}{2}i$$

**Q5: Perform the indicated operations and reduce to the form $\boldsymbol{a + bi}$**

**(i)** $\boldsymbol{(4 - 3i) + (2 - 3i)}$

**Solution:**

$(4 - 3i) + (2 - 3i)$
$= 4 - 3i + 2 - 3i$
$= 4 + 2 - 3i - 3i$
$= 6 - 6i$



**(ii)** $\boldsymbol{(5 - 2i) - (4 - 7i)}$

**Solution:**

$(5 - 2i) - (4 - 7i)$
$= 5 - 2i - 4 + 7i$
$= 5 - 4 - 2i + 7i$
$= 1 + 5i$

**(iii)** $\boldsymbol{2i(4 - 5i)}$

**Solution:**

$2i(4 - 5i)$
$= 2i - 10i^2$
$= 2i - 10(-1)$
$= 2i + 10$
$= 10 + 2i$

### Ex # 2.5

**(iv)** $(2-3i)\div(4-5i)$

**Solution:**

$(2-3i)\div(4-5i)$

$=\dfrac{2-3i}{4-5i}$

Multiply and divide by $4+5i$

$=\dfrac{2-3i}{4-5i}\times\dfrac{4+5i}{4+5i}$

$=\dfrac{(2-3i)(4+5i)}{(4-5i)(4+5i)}$

$=\dfrac{8+10i-12i-15i^2}{(4)^2-(5i)^2}$

$=\dfrac{8-2i-15(-1)}{16-25i^2}$

$=\dfrac{8-2i+15}{16-25(-1)}$

$=\dfrac{8+15-2i}{16+25}$

$=\dfrac{23-2i}{41}$

$=\dfrac{23}{41}-\dfrac{2}{41}i$

**Q6: Find the complex conjugate of the following complex numbers.**

**(i)** $-8-3i$

The complex conjugate of $-8-3i$ is $-8+3i$

**(i)** $-4+9i$

The complex conjugate of $-4+9i$ is $-4-9i$

**(iii)** $7+6i$

The complex conjugate of $7+6i$ is $7-6i$

**(iv)** $\sqrt{5}-i$

The complex conjugate of $\sqrt{5}-i$ is $\sqrt{5}+i$

## Review Ex # 2

**Page # 73**

**Q3: Simplify each of the following.**

**(i)** $\left(\dfrac{-2}{3}\right)^3$

**Solution:**

$\left(\dfrac{-2}{3}\right)^3$

$=\dfrac{(-2)^3}{(3)^3}$

$=\dfrac{-8}{27}$

**(ii)** $(-2)^3.(3)^2$

**Solution:**

$(-2)^3.(3)^2$

$=-8\times 9$

$-72$

**(iii)** $-3\sqrt{48}$

**Solution:**

$-3\sqrt{48}$

$-3\sqrt{4\times 4\times 3}$

$-3\sqrt{4\times 4}\times\sqrt{3}$

$-3\times 4\sqrt{3}$

$-12\sqrt{3}$

**(iv)** $\dfrac{5}{\sqrt[3]{9}}$

**Solution:**

$\dfrac{5}{\sqrt[3]{9}}$

$=\dfrac{5}{(9)^{\frac{1}{3}}}$

$=\dfrac{5}{(3^2)^{\frac{1}{3}}}$

$=\dfrac{5}{(3)^{\frac{2}{3}}}$

**Review Ex # 2**

**Multiply and Divide by $\sqrt[3]{3}$**

$$\frac{5}{(3)^{\frac{2}{3}}} \times \frac{\sqrt[3]{3}}{\sqrt[3]{3}}$$

$$\frac{5 \times \sqrt[3]{3}}{(3)^{\frac{2}{3}} \times (3)^{\frac{3}{3}}}$$

$$\frac{5\sqrt[3]{3}}{(3)^{\frac{2}{3}+\frac{3}{3}}}$$

$$\frac{5\sqrt[3]{3}}{(3)^{\frac{3}{3}}}$$

$$\frac{5\sqrt[3]{3}}{3}$$

**Q4:** **Multiply $8i, \ -8i$**

**Solution:**

$8i, \ -8i$

Now

$(8i)(-8i) = -64i^2$

$= -64(-1)$

$= 64$

**Q5:** **Divide $2-5i$ by $1-6i$**

**Solution:**

$$\frac{2-5i}{1-6i}$$

$i$

**Multiply and divide by $1+6i$**

$$= \frac{2-5i}{1-6} \times \frac{1+6i}{1+6i}$$

$$= \frac{(2-5i)(1+6i)}{(1-6i)(1+6i)}$$

$$= \frac{2+12i-5i-30i^2}{(1)^2-(6i)^2}$$

$$= \frac{2+7i-30(-1)}{1-36i^2}$$

$$= \frac{2+7i+30}{1-36(-1)}$$

**Review Ex # 2**

$$= \frac{2+30+7i}{1+36}$$

$$= \frac{32+7i}{37}$$

$$= \frac{32}{37} - \frac{7}{37}i$$

**Q7:** **Use laws of exponents to simplify:**

$$\frac{(\mathbf{81})^{n}.\mathbf{3^5} + (\mathbf{3})^{4n-1}(\mathbf{243})}{(\mathbf{9^{2n}})(\mathbf{3^3})}$$

**Solution:**

$$\frac{(81)^n.3^5 + (3)^{4n-1}(243)}{(9^{2n})(3^3)}$$

$$= \frac{(3^4)^n.3^5 + 3^{4n-1}.(3^5)}{(3^2)^{2n}(3^3)}$$

$$= \frac{3^{4n}.3^5 + 3^{4n}.3^{-1}.3^5}{3^{4n}.3^3}$$

$$= \frac{3^{4n}.3^5(1+3^{-1})}{3^{4n}.3^3}$$

$$= \frac{3^{4n}.3^3.3^2(1+3^{-1})}{3^{4n}.3^3}$$

$$= 3^2(1+3^{-1})$$

$$= 9\left(1+\frac{1}{3}\right)$$

$$= 9\left(\frac{3+1}{3}\right)$$

$$= 9\left(\frac{4}{3}\right)$$

$$= 3 \times 4$$

$$= 12$$

**Q6:** **Name the property used**

$$\mathbf{7 \times \frac{1}{7} = \frac{1}{7} \times 7 = 1}$$

**Answer:**

**Multiplicative Property**

# MATHEMATICS

## Class 9th (KPK)

NAME: ______________________________

F.NAME: _____________________________

CLASS:_____________ SECTION: _________

ROLL #: ______ SUBJECT: ______________

ADDRESS: _________________________________________

_________________________________________________

SCHOOL: ___________________________________________

# UNIT # 3
# LOGARITHM

## Exercise # 3.1

**SCIENTIFIC NOTATION:**

Scientific notation is a way of writing numbers that are too big or too small to be easily written in decimal form.

**Representation**

The positive number "$x$" is represented in scientific notation as the product of two numbers where the first number "$a$" is a real number greater than 1 and less than 10 and the second is the integral power of "$n$" of 10.

$$x = a \times 10^n$$

**Rules for Standard Notation to Scientific Notation**

(i) In a given number, place the decimal after first non-zero digit.

(ii) If the decimal point is moved towards left, then the power of "10" will be positive.

(iii) If the decimal is moved towards right, then the power of "10" will be negative.
The numbers of digits through which the decimal point has been moved will be the exponent.

**Rules for Standard Notation to Scientific Notation**

(i) If the exponent of 10 is Positive, move the decimal towards Right.

(ii) If the exponent of 10 is Negative, move the decimal toward Left.

(iii) Move the decimal point to the same number of digits as the exponent of 10.

**Example # 7 Page # 80**

How many miles does light travel in 1 day? The speed of the light is 186,000 mi/ sec. write the answer in scientific notation.

**Solution:**

$$Time = t = 1\ day = 24\ hr$$
$$t = 24 \times 60 \times 60\ sec = 86400$$
$$t = 8.64 \times 10^4\ sec$$
$$Speed = v = 186000\ mi/sec$$
$$v = 1.86 \times 10^5\ mi/sec$$

As we know that

$$s = vt$$

Put the values

$$s = 1.86 \times 10^5 \times 8.64 \times 10^4$$
$$s = 1.86 \times 8.64 \times 10^5 \times 10^4$$
$$s = 16.0704 \times 10^{5+4}$$
$$s = 16.0704 \times 10^9$$
$$s = 1.60704 \times 10^1 \times 10^9$$
$$s = 1.60704 \times 10^{10}$$

Thus light travels $1.60704 \times 10^1 \times 10^9$ miles in a day

## Exercise # 3.1

**Page # 80**

**Q1: Write each number in scientific notation.**

**(i) 405,000**

**Solution:**

405,000

**In Scientific Form:**

$$4.05 \times 10^4$$

**(ii) 1,670,000**

**Solution:**

1,670,000

**In Scientific Form:**

$$1.67 \times 10^6$$

**(iii) 0.00000039**

**Solution:**

0.00000039

**In Scientific Form:**

$$3.9 \times 10^{-7}$$

**(iv) 0.00092**

**Solution:**

0.00092

**In Scientific Form:**

$$9.2 \times 10^{-4}$$

**Ex # 3.1**

**(v) 234,600,000,000**

**Solution:**

234,600,000,000

**In Scientific Form:**

$$2.346 \times 10^{11}$$

**(vi) 8,904,000,000**

**Solution:**

8,904,000,000

**In Scientific Form:**

$$8.904 \times 10^{9}$$

**(vii) 0.00104**

**Solution:**

0.00104

**In Scientific Form:**

$1.04 \times 10^{-3}$

**(viii) 0.00000000514**

**Solution:**

0.00000000514

**In Scientific Form:**

$$5.14 \times 10^{-9}$$

**(ix) $\mathbf{0.05 \times 10^{-3}}$**

**Solution:**

$$0.05 \times 10^{-3}$$

**In Scientific Form:**

$$5.0 \times 10^{-2} \times 10^{-3}$$
$$5.0 \times 10^{-2-3}$$
$$5.0 \times 10^{-5}$$

**Q2: Write each number in standard notation.**

**(i) $\mathbf{8.3 \times 10^{-5}}$**

**Solution:**

$8.3 \times 10^{-5}$

**In Standard Form:**

0.000083

**(ii) $\mathbf{4.1 \times 10^{6}}$**

**Solution:**

$4.1 \times 10^{6}$

**In Standard Form:**

410000

**Ex # 3.1**

**(iii) $\mathbf{2.07 \times 10^{7}}$**

**Solution:**

$2.07 \times 10^{7}$

**In Standard Form:**

20700000

**(iv) $\mathbf{3.15 \times 10^{-6}}$**

**Solution:**

$3.15 \times 10^{-6}$

**In Standard Form:**

0.00000315

**(v) $\mathbf{6.27 \times 10^{-10}}$**

**Solution:**

$6.27 \times 10^{-10}$

**In Standard Form:**

0.000000000627

**(vi) $\mathbf{5.41 \times 10^{-8}}$**

**Solution:**

$5.41 \times 10^{-8}$

**In Standard Form:**

0.0000000541

**(vii) $\mathbf{7.632 \times 10^{-4}}$**

**Solution:**

$7.632 \times 10^{-4}$

**In Standard Form:**

0.0007632

**(viii) $\mathbf{9.4 \times 10^{5}}$**

**Solution:**

$9.4 \times 10^{5}$

**In Standard Form:**

940000

**(ix) $\mathbf{-2.6 \times 10^{9}}$**

**Solution:**

$-2.6 \times 10^{9}$

**In Standard Form:**

$-2600000000$

**Ex # 3.1**

**Q3: How long does it take light to travel to Earth from the sun? The sun is $9.3 \times 10^7$ miles from Earth, and light travels $1.86 \times 10^5$ mi/s.**

**Solution:**

Given:

Distance between earth and sun $= 9.3 \times 10^7$ miles

Speed of light $= 1.86 \times 10^5$ mi/s

As we have:

$s = vt$

$\frac{s}{v} = t$

Or

$t = \frac{s}{v}$

Put the values:

$t = \frac{9.3 \times 10^7}{1.86 \times 10^5}$

$t = 5 \times 10^7 \times 10^{-5}$

$t = 5 \times 10^{7-5}$

$t = 5 \times 10^2$

$t = 500 \ sec$

$t = 480 \ \text{sec} + 20 \ sec$

$t = 8 \ \text{min} \ 20 \ sec$

**Exercise # 3.2**

**Logarithm**

If $a^x = y$ then the index $x$ is called the logarithm of $y$ to the base $a$ and written as $\log_a y = x$.

We called $\log_a y = x$ like log of y to the base a equal to $x$.

| Logarithm Form | Exponential Form |
|---|---|
| $\log_a y = x$ | $a^x = y$ |
| $\log_8 64 = 2$ | $8^2 = 64$ |

**Ex # 3.2**

**Page # 83**

**Q1: Write the following in logarithm form.**

**(i) $4^4 = 256$**

**Solution:**

$4^4 = 256$

**In logarithm form**

$\log_4 256 = 4$

**(ii) $2^{-6} = \frac{1}{64}$**

**Solution:**

$2^{-6} = \frac{1}{64}$

**In logarithm form**

$\log_2 \frac{1}{64} = -6$

**(iii) $10^0 = 1$**

**Solution:**

$10^0 = 1$

**In logarithm form**

$\log_{10} 1 = 0$

**(iv) $x^{\frac{3}{4}} = y$**

**Solution:**

$x^{\frac{3}{4}} = y$

**In logarithm form**

$\log_x y = \frac{3}{4}$

**(v) $3^{-4} = \frac{1}{81}$**

**Solution:**

$3^{-4} = \frac{1}{81}$

**In logarithm form**

$\log_3 \frac{1}{81} = -4$

**(vi) $64^{\frac{2}{3}} = 16$**

**Solution:**

$64^{\frac{2}{3}} = 16$

**In logarithm form**

$\log_{64} 16 = \frac{2}{3}$

**Ex # 3.2**

**Q2: Write the following in exponential form.**

**(i)** $\log_a\left(\frac{1}{a^2}\right) = -1$

**Solution:**

$\log_a\left(\frac{1}{a^2}\right) = -1$

**In exponential form**

$a^{-1} = \frac{1}{a^2}$

**(ii)** $\log_2 \frac{1}{128} = -7$

**Solution:**

$\log_2 \frac{1}{128} = -7$

**In exponential form**

$2^{-7} = \frac{1}{128}$

**(iii)** $\log_b 3 = 64$

**Solution:**

$\log_b 3 = 64$

**In exponential form**

$b^{64} = 3$

**(iv)** $\log_a a = 1$

**Solution:**

$\log_a a = 1$

**In exponential form**

$a^1 = 1$

**(v)** $\log_a 1 = 0$

**Solution:**

$\log_a 1 = 0$

**In exponential form**

$a^0 = 1$

**(vi)** $\log_4 \frac{1}{8} = \frac{-3}{2}$

**Solution:**

$\log_4 \frac{1}{8} = \frac{-3}{2}$

**In exponential form**

$4^{\frac{-3}{2}} = \frac{1}{8}$

**Ex # 3.2**

**Q3: Solve:**

**(i)** $\log_{\sqrt{5}} 125 = x$

**Solution:**

$\log_{\sqrt{5}} 125 = x$

**In exponential form**

$(\sqrt{5})^x = 125$

$\left(5^{\frac{1}{2}}\right)^x = 5 \times 5 \times 5$

$5^{\frac{x}{2}} = 5^3$

Now

$\frac{x}{2} = 3$

**Multiply B.S by 2**

$2 \times \frac{x}{2} = 2 \times 3$

$x = 6$

**(ii)** $\log_4 x = -3$

**Solution:**

$\log_4 x = -3$

**In exponential form**

$4^{-3} = x$

**Now**

$\frac{1}{4^3} = x$

$\frac{1}{4 \times 4 \times 4} = x$

$\frac{1}{64} = x$

**Or**

$x = \frac{1}{64}$

**(iii)** $\log_{81} 9 = x$

**Solution:**

$\log_{81} 9 = x$

**In exponential form**

$81^x = 9$

$(9^2)^x = 9^1$

$9^{2x} = 9^1$

Now $2x = 1$

**Divide B.S by 2**

$\frac{2x}{2} = \frac{1}{2}$

$2x = \frac{1}{2}$

**Ex # 3.2**

**(iv)** $\log_3(5x+1)=2$

**Solution:**

$\log_3(5x+1)=2$

**In exponential form**

$3^2=5x+1$

$9=5x+1$

**Subtract 1 form B.S**

$9-1=5x+1-1$

$8=5x$

**Divide B.S by 5**

$\frac{8}{5}=\frac{5x}{5}$

$\frac{8}{5}=x$

$x=\frac{8}{5}$

**(v)** $\log_2 x=7$

**Solution:**

$\log_2 x=7$

**In exponential form**

$2^7=x$

**Now**

$2\times2\times2\times2\times2\times2\times2=x$

$128=x$

$x=128$

**(vi)** $\log_x 0.25=2$

**Solution:**

$\log_x 0.25=2$

**In exponential form**

$x^2=0.25$

$x^2=\frac{25}{100}$

**Taking square root on B.S**

$\sqrt{x^2}=\sqrt{\frac{25}{100}}$

$x=\frac{5}{10}$

$x=\frac{1}{2}$

**Ex # 3.2**

**(vii)** $\log_x(0.001)=-3$

**Solution:**

$\log_x(0.001)=-3$

**In exponential form**

$x^{-3}=0.001$

$x^{-3}=\frac{1}{1000}$

$x^{-3}=\frac{1}{10^3}$

$x^{-3}=10^{-3}$

**So**

$x=10$

**(viii)** $\log_x\frac{1}{64}=-2$

**Solution:**

$\log_x\frac{1}{64}=-2$

**In exponential form**

$x^{-2}=\frac{1}{64}$

$x^{-2}=\frac{1}{8\times8}$

$x^{-2}=\frac{1}{8^2}$

$x^{-2}=8^{-2}$

**So**

$x=8$

**(ix)** $\log_{\sqrt{3}} x=16$

**Solution:**

$\log_{\sqrt{3}} x=16$

**In exponential form**

$\left(\sqrt{3}\right)^{16}=x$

$\left(3^{\frac{1}{2}}\right)^{16}=x$

$3^{\frac{16}{2}}=x$

$3^8=x$

$3\times3\times3\times3\times3\times3\times3\times3=x$

$6561=x$

$x=6561$

**Exercise # 3.3**

**COMMON LOGARITHM**

**Introduction**

The common logarithm was invented by a British Mathematician Prof. Henry Briggs (1560-1631).

**Definition**

Logarithms having base 10 are called common logarithms or Briggs logarithms.

**Note:**

The base of logarithm is not written because it is considered to be 10.

Logarithm of the number consists of two parts. Characteristics and Mantissa

**Example:** 1.5377

**Characteristics**

The digit before the decimal point or Integral part is called characteristics

**Mantissa**

The decimal fraction part is mantissa.

In above example

1 is Characteristics and .5377 is Mantissa.

**USE OF LOG TABLE TO FIND MANTISSA:**

**A logarithm table is divided into three parts.**

**(i)** The first part of the table is the extreme left column contains number from10 to 99.

**(ii)** The second part of the table consists of 10 columns headed by 0, 1, 2, …. 9. The number under these columns are taken to find mantissa.

**(iii)** The third part consists of small columns known as mean difference headed by 1, 2, 3, … 9. These columns are added to the Mantissa found in second column.

**To Find Mantissa**

Let we have an example: 763.5

**Solution:**

**(i)** First ignore the decimal point.

**(ii)** Take first two digits e.g. 76 and proceed along this row until we come to column headed by third digit 3 of the number which is 8825.

**(iii)** Now take fourth digit i.e. 5 and proceed along this row in mean difference column which is 5.

**(iv)** Now add $8825 + 3 = 8828$

# Ex # 3.3

**Page # 86**

**Q1: Find the characteristics of the common logarithm of each of the following numbers.**

**(i) 57**

**In Scientific form:**

$5.7 \times 10^1$

Thus Characteristics $= 1$

**(ii) 7.4**

**In Scientific form:**

$7.4 \times 10^0$

Thus Characteristics $= 0$

**(iii) 5.63**

**In Scientific form:**

$5.63 \times 10^0$

Thus Characteristics $= 0$

**(iv) 56.3**

**In Scientific form:**

$5.63 \times 10^1$

Thus Characteristics $= 1$

**(v) 982.5**

**In Scientific form:**

$9.825 \times 10^2$

Thus Characteristics $= 2$

**(vi) 7824**

**In Scientific form:**

$7.824 \times 10^3$

Thus Characteristics $= 3$

**(vii) 186000**

**In Scientific form:**

$1.86 \times 10^5$

Thus Characteristics $= 5$

**viii. 0.71**

**In Scientific form:**

$7.1 \times 10^{-1}$

Thus Characteristics $= -1$

**Ex # 3.3**

**Q2: Find the following.**

**(i) $\log 87.2$**

**Solution:**

$\log 87.2$

**In Scientific form:**

$8.72 \times 10^1$

Thus Characteristics $= 1$

To find Mantissa, using Log Table:

So Mantissa $= .9405$

Hence $\log 87.2 = 1.9405$

**(ii) $\log 37300$**

**Solution:**

$\log 37300$

**In Scientific form:**

$3.73 \times 10^4$

Thus Characteristics $= 4$

To find Mantissa, using Log Table:

So Mantissa $= .5717$

Hence $\log 37300 = 4.5717$

**(iii) $\log 753$**

**Solution:**

$\log 753$

**In Scientific form:**

$7.53 \times 10^2$

Thus Characteristics $= 2$

To find Mantissa, using Log Table:

So Mantissa $= .8768$

Hence $\log 753 = 2.8768$

**(iv) $\log 9.21$**

**Solution:**

$\log 9.21$

**In Scientific form:**

$9.21 \times 10^0$

Thus Characteristics $= 0$

To find Mantissa, using Log Table:

So Mantissa $= .9643$

Hence $\log 9.21 = 0.9643$

**Ex # 3.3**

**(v) $\log 0.00159$**

**Solution:**

$\log 0.00159$

**In Scientific form:**

$1.59 \times 10^{-3}$

Thus Characteristics $= -3$

To find Mantissa, using Log Table:

So Mantissa $= .2014$

Hence $\log 0.00159 = \bar{3}.2014$

**(vi) $\log 0.0256$**

**Solution:**

$\log 0.0256$

**In Scientific form:**

$2.56 \times 10^{-2}$

Thus Characteristics $= -2$

To find Mantissa, using Log Table:

So Mantissa $= .4082$

Hence $\log 0.0256 = \bar{2}.4082$

**(vii) $\log 6.753$**

**Solution:**

$\log 6.753$

**In Scientific form:**

$6.753 \times 10^0$

Thus Characteristics $= 0$

**To find Mantissa, using Log Table**

Mantissa $= .8295$

Hence $\log 6.753 = 0.8295$

| $R.W$ |
|---|
| $8293 + 2$<br>$= 8295$ |

**Q3: Find logarithms of the following numbers.**

**(i) 2476**

**Solution:**

2476

$Let\ x = 2476$

Taking log on B.S

$\log x = \log 2476$

**In Scientific form:**

$2.476 \times 10^3$

Thus Characteristics $= 3$

**To find Mantissa, using Log Table**

So Mantissa $= .3927 + 11$

Mantissa $= .3938$

Hence $\log 2476 = 3.3938$

| $R.W$ |
|---|
| $3927 + 11$<br>$= 3938$ |

**Ex # 3.3**

**(ii) 2.4**

**Solution:**

2.4

$Let\ x = 2.4$

Taking log on B.S

$\log x = \log 2.4$

**In Scientific form:**

$2.4 \times 10^0$

Thus Characteristics $= 0$

**To find Mantissa, using Log Table:**

So Mantissa $= .3802$

Hence $\log 2.4 = 0.3802$

**(iii) 92.5**

**Solution:**

92.5

$Let\ x = 92.5$

Taking log on B.S

$\log x = \log 92.5$

**In Scientific form:**

$9.25 \times 10^1$

Thus Characteristics $= 1$

**To find Mantissa, using Log Table:**

So Mantissa $= .9661$

Hence $\log 92.5 = 1.9661$

**(iv) 482.7**

**Solution:**

482.7

$Let\ x = 482.7$

Taking log on B.S

$\log x = \log 482.7$

**In Scientific form:**

$4.827 \times 10^2$

Thus Characteristics $= 2$

**To find Mantissa, using Log Table:**

So Mantissa $= .6836$

Hence $\log 482.7 = 2.6836$

| $R.W$ |
|---|
| $6830 + 6$<br>$= 6836$ |

**Ex # 3.3**

**(v) 0.783**

**Solution:**

0.783

$Let\ x = 0.783$

Taking log on B.S

$\log x = \log 0.783$

**In Scientific form:**

$7.83 \times 10^{-1}$

Thus Characteristics $= -1$

**To find Mantissa, using Log Table:**

So Mantissa $= .8938$

Hence $\log 0.783 = \bar{1}.8938$

**(vi) 0.09566**

**Solution:**

0.09566

$Let\ x = 0.09566$

Taking log on B.S

$\log x = \log 0.09566$

**In Scientific form:**

$9.566 \times 10^{-2}$

Thus Characteristics $= -2$

To find Mantissa, using Log Table:

So Mantissa $= .9808$

Hence $\log 0.09566 = \bar{2}.9808$

| $R.W$ |
|---|
| $9805 + 3$<br>$= 9808$ |

**(vii) 0.006753**

**Solution:**

0.006753

$Let\ x = 0.006753$

Taking log on B.S

$\log x = \log 0.006753$

**In Scientific form:**

$6.753 \times 10^{-3}$

Thus Characteristics $= -3$

To find Mantissa, using Log Table:

So Mantissa $= .8295$

Hence $\log 0.006735 = \bar{3}.8295$

| $R.W$ |
|---|
| $8293 + 2$<br>$= 8295$ |

Ex # 3.3

**(viii) 700**

**Solution:**

700

$Let\ x = 700$

Taking log on B.S

$\log x = \log 700$

**In Scientific form:**

$7.00 \times 10^2$

Thus Characteristics = 2

To find Mantissa, using Log Table:

So Mantissa = .8451

Hence $\log 700 = 2.8451$

## Exercise # 3.4

**ANTI-LOGARITHM**

If $\log x = y$ then $x$ is the anti-logarithm of y and written as $x = anti - \log y$

**Explanation with Example:**

2.3456

(i) Here the digit before decimal point is Characteristics i.e. 2

(ii) And Mantissa= .3456

**To find anti-log, we see Mantissa in Anti-log Table**

(i) Take first two digits i.e. .34 and proceed along this row until we come to column headed by third digit 5 of the number which is 2213.

(ii) Now take fourth digit i.e. 6 and proceed along this row which is 3.

(iii) Now add $2213 + 3 = 2216$

So to find anti-log, write it in Scientific form like

$anti - \log 2.3456 = 2.2216 \times 10^{char}$

$anti - \log 2.3456 = 2.216 \times 10^2$

$anti - \log 2.3456 = 221.6$

# Ex # 3.4

**Page # 88**

**Q1: Find anti-logarithm of the following numbers.**

**(i) 1.2508**

**Solution:**

1.2508

$Let\ \log x = 1.2508$

**Taking anti-log on B.S**

$Anti - \log(\log x) = \text{Anti} - \log 1.2508$

$x = \text{Anti} - \log 1.2508$

Characteristics = 1

Mantissa = .2508

| $R.W$ |
| --- |
| 1778+3<br>= 1781 |

So

$x = 1.781 \times 10^1$

$x = 17.81$

**(ii) 0.8401**

**Solution:**

0.8401

$Let\ \log x = 0.8401$

**Taking anti-log on B.S**

$Anti - \log(\log x) = \text{Anti} - \log 0.8401$

$x = \text{Anti} - \log 0.8401$

Characteristics = 0

Mantissa = .8401

| $R.W$ |
| --- |
| 6918+2<br>= 6920 |

So

$x = 6.920 \times 10^0$

$x = 6.920$

**(iii) 2.540**

**Solution:**

2.540

$Let\ \log x = 2.540$

**Taking anti-log on B.S**

$Anti - \log(\log x) = \text{Anti} - \log 2.540$

$x = \text{Anti} - \log 2.540$

Characteristics = 2

Mantissa = .540

So

$x = 3.467 \times 10^2$

$x = 346.7$

**Ex # 3.4**

**(iv)** $\bar{2}.2508$

**Solution:**

$\bar{2}.2508$

$Let\ \log x = \bar{2}.2508$

**Taking anti-log on B.S**

$Anti - \log(\log x) = \text{Anti} - \log \bar{2}.2508$

$x = \text{Anti} - \log \bar{2}.2508$

Characteristics $= -2$

Mantissa $= .2508$

So

$x = 1.781 \times 10^{-2}$

$x = 0.01781$

| $R.W$ |
|---|
| 1778+3<br>= 1781 |

**(v)** $\bar{1}.5463$

**Solution:**

$\bar{1}.5463$

$Let\ \log x = \bar{1}.5463$

**Taking anti-log on B.S**

$Anti - \log(\log x) = \text{Anti} - \log \bar{1}.5463$

$x = \text{Anti} - \log \bar{1}.5463$

Characteristics $= -1$

Mantissa $= .5463$

So

$x = 3.518 \times 10^{-1}$

$x = 0.3518$

| $R.W$ |
|---|
| 3516+2<br>= 3518 |

**(vi)** **3.5526**

**Solution:**

3.5526

$Let\ \log x = 3.5526$

**Taking anti-log on B.S**

$Anti - \log(\log x) = \text{Anti} - \log 3.5526$

$x = \text{Anti} - \log 3.5526$

Characteristics $= 3$

Mantissa $= .5526$

So

$x = 3.570 \times 10^{3}$

$x = 3570$

| $R.W$ |
|---|
| 3565+5<br>= 3570 |

**Ex # 3.4**

**Q2:** **Find the values of $x$ from the following equations:**

**(i)** $\log x = \bar{1}.8401$

**Solution:**

$\log x = \bar{1}.8401$

**Taking anti $-$ log on B.S**

$\text{Anti} - \log(\log x) = \text{Anti} - \log \bar{1}.8401$

$x = \text{Anti} - \log \bar{1}.8401$

Characteristics $= -1$

Mantissa $= .8401$

So

$x = 6.920 \times 10^{-1}$

$x = 0.6920$

| $R.W$ |
|---|
| $6918 + 2$<br>$= 6920$ |

**(ii)** $\log x = 2.1931$

**Solution:**

$\log x = 2.1931$

**Taking anti $-$ log on B.S**

$\text{Anti} - \log(\log x) = \text{Anti} - \log 2.1931$

$x = \text{Anti} - \log 2.1931$

Characteristics $= 2$

Mantissa $= .1931$

So

$x = 1.560 \times 10^{2}$

$x = 156.0$

| $R.W$ |
|---|
| $1560 + 0$<br>$= 1560$ |

**(iii)** $\log x = 4.5911$

**Solution:**

$\log x = 4.5911$

**Taking anti $-$ log on B.S**

$\text{Anti} - \log(\log x) = \text{Anti} - \log 4.5911$

$x = \text{Anti} - \log 4.5911$

Characteristics $= 4$

Mantissa $= .5911$

So

$x = 3.900 \times 10^{4}$

$x = 39000.0$

| $R.W$ |
|---|
| $3899 + 1$<br>$= 3900$ |

## Ex # 3.4

**(i)** $\log x = \bar{3}.0253$

**Solution:**

$\log x = \bar{3}.0253$

**Taking anti – log on B.S**

$\text{Anti}-\log(\log x) = \text{Anti}-\log \bar{3}.0253$

$x = \text{Anti}-\log \bar{3}.0253$

Characteristics $= -3$

Mantissa $= .0253$

So

$x = 1.060 \times 10^{-3}$

$x = 0.001060$

| $R.W$ |
|---|
| $1059 + 1$ <br> $= 1060$ |

**(ii)** $\log x = 1.8716$

**Solution:**

$\log x = 1.8716$

**Taking anti – log on B.S**

$\text{Anti}-\log(\log x) = \text{Anti}-\log 1.8716$

$x = \text{Anti}-\log 1.8716$

Characteristics $= 1$

Mantissa $= .8716$

So

$x = 7.440 \times 10^{1}$

$x = 74.40$

| $R.W$ |
|---|
| $7430 + 10$ <br> $= 7440$ |

**(iii)** $\log x = \bar{2}.8370$

**Solution:**

$\log x = \bar{2}.8370$

**Taking anti – log on B.S**

$\text{Anti}-\log(\log x) = \text{Anti}-\log \bar{2}.8370$

$x = \text{Anti}-\log \bar{2}.8370$

Characteristics $= -2$

Mantissa $= .8370$

So

$x = 6.871 \times 10^{-2}$

$x = 0.06781$

## Ex # 3.5

**LAWS OF LOGARITHM**

**(i)** $\log_a mn = \log_a m + \log_a n$

**or** $\log mn = \log m + \log n$

**Example:**

$\log 2 \times 3 = \log 2 + \log 3$

**(ii)** $\log_a \frac{m}{n} = \log_a m - \log_a n$

**or** $\log \frac{m}{n} = \log m - \log n$

**Example:**

$\log \frac{3}{5} = \log 3 - \log 5$

$\log 6 - \log 3 = \log \frac{6}{3} = \log 2$

**(iii)** $\log_a m^n = n \log_a m$

**or** $\log m^n = n \log m$

**Example:**

$\log 2^3 = 3 \log 2$

$\log_a m \log_m n = \log_a n$

$\log_2 3 \log_3 5 = \log_3 5$

$\log_m n = \dfrac{\log_a n}{\log_a m}$

**Example:**

**(iv)** $\dfrac{\log_7 r}{\log_7 t} = \log_t r$

**Note:**

**(i)** $\log_a a = 1$

**(ii)** $\log_{10} 10 = 1$

**(iii)** $\log 10 = 1$

**(iv)** $\log_{10} 1 = 0$

**(v)** $\log 1 = 0$

**(vi)** $\log_m n = \dfrac{\log_a n}{\log_a m}$

This is called Change of Base Law

**Ex # 3.5**

**Proof of Laws of Logarithm one by one**

(i) $\log_a mn = \log_a m + \log_a n$

**Proof:**

$Let\ \ \log_a m = x\ \ and\ \ \log_a n = y$

Write them in Exponential form:

$a^x = m\ \ and\ \ a^y = n$

Now multiply these:

$a^x \times a^y = mn$

Or $mn = a^x \times a^y$

$mn = a^{x+y}$

Taking $\log_a$ on B.S

$\log_a mn = \log_a a^{x+y}$

$\log_a mn = (x+y)\log_a a$

$\log_a mn = (x+y)(1)$ $\quad\therefore \log_a a = 1$

$\log_a mn = x + y$

$\log_a mn = \log_a m + \log_a n$

(ii) $\log_a \dfrac{m}{n} = \log_a m - \log_a n$

**Proof:**

$Let\ \ \log_a m = x\ \ and\ \ \log_a n = y$

Write them in Exponential form:

$a^x = m\ \ and\ \ a^y = n$

Now Divide these:

$\dfrac{a^x}{a^y} = \dfrac{m}{n}$

Or

$\dfrac{m}{n} = \dfrac{a^x}{a^y}$

$\dfrac{m}{n} = a^{x-y}$

Taking $\log_a$ on B.S

$\log_a \dfrac{m}{n} = \log_a a^{x-y}$

$\log_a \dfrac{m}{n} = (x-y)\log_a a$

$\log_a \dfrac{m}{n} = (x-y)(1)$ $\quad\therefore \log_a a = 1$

$\log_a \dfrac{m}{n} = x - y$

$Hence\ \ \log_a \dfrac{m}{n} = \log_a m - \log_a n$

**Ex # 3.5**

(iii) $\log_a m^n = n\log_a m$

**Proof:**

$Let\ \ \log_a m = x$

In Exponential form:

$a^x = m$

Or

$m = a^x$

Taking power 'n' on B.S

$m^n = (a^x)^n$

$m^n = a^{nx}$

Taking $\log_a$ on B.S

$\log_a m^n = \log_a a^{nx}$

$\log_a m^n = nx\log_a a$

$\log_a m^n = nx(1)$ $\quad\therefore \log_a a = 1$

$\log_a m^n = nx$

$\log_a m^n = n\log_a m$

(iv) $\log_a m \log_m n = \log_a n$

**Proof:**

$Let\ \ \log_a m = x\ \ and\ \ \log_m n = y$

Write them in Exponential form:

$a^x = m\ \ and\ \ m^y = n$

Now multiply these:

$As\ a^{xy} = (a^x)^y$

$But\ \ (a^x)^y = m$

$So\ a^{xy} = (m)^y = n$

$So\ a^{xy} = n$

Taking $\log_a$ on B.S

$\log_a a^{xy} = \log_a n$

$(xy)\log_a a = \log_a n$

$xy(1) = \log_a n$ $\quad\therefore \log_a a = 1$

Now

$\log_a m \log_m n = \log_a n$

**Example # 14 page # 90**

$-1 + \log y$

**Solution:**

$= -1 + \log y$

$= -\log 10 + \log y$

$= \log 10^{-1} + \log y$

$= \log \dfrac{1}{10} + \log y$

$= \log 0.1 + \log y$

$= \log 0.1\,y$

# Ex # 3.5

**Page # 91**

**Q1: Use logarithm properties to simplify the expression.**

**(i)** $\log_7 \sqrt{7}$

**Solution:**

$\log_7 \sqrt{7}$

$Let\ x = \log_7 \sqrt{7}$

$x = \log_7 (7)^{\frac{1}{2}}$

$As\ \log_a m^n = n \log_a m$

$x = \frac{1}{2}\log_7 7$

$x = \frac{1}{2}(1) \qquad \therefore \log_a a = 1$

$x = \frac{1}{2}$

**(ii)** $\log_8 \frac{1}{2}$ Trick

**Solution:**

$\log_8 \frac{1}{2}$

$\log_8 \frac{1}{2}$

$Let\ \ \log_8 \frac{1}{2} = x$

**In exponential form:**

$8^x = \frac{1}{2}$

$(2^3)^x = 2^{-1}$

$2^{3x} = 2^{-1}$

Now

$3x = -1$

Divide B.S by 3, we get

$x = \frac{-1}{3}$

**(iii)** $\log_{10} \sqrt{1000}$

**Solution:**

$\log_{10} \sqrt{1000}$

$Let\ x = \log_{10} (10^3)^{\frac{1}{2}}$

$x = \log_{10} (10)^{\frac{3}{2}}$



$As\ \log_a m^n = n \log_a m$

$x = \frac{3}{2}\log_{10} 10$

$x = \frac{3}{2}(1) \qquad \therefore \log_a a = 1$

$x = \frac{3}{2}$

**(iv)** $\log_9 3 + \log_9 27$

**Solution:**

$\log_9 3 + \log_9 27$

$Let\ x = \log_9 3 + \log_9 27$

$As\ \log_a mn = \log_a m + \log_a n$

$x = \log_9 3 \times 27$

$x = \log_9 81$

$x = \log_9 9^2$

$As\ \log_a m^n = n \log_a m$

$x = 2 \log_9 9$

$x = 2(1) \qquad \therefore \log_a a = 1$

$x = 2$

**(v)** $\log \frac{1}{(0.0035)^{-4}}$

**Solution:**

$\log \frac{1}{(0.0035)^{-4}}$

$Let\ \ x = \log \frac{1}{(0.0035)^{-4}}$

$As\ \log_a \frac{m}{n} = \log_a m - \log_a n$

$x = \log 1 - \log(0.0035)^{-4}$

**As** $\log 1 = 0$ **and** $\log_a m^n = n \log_a m$

**Thus**

$x = 0 - (-4)\log 0.0035$

**Here** $Ch = -3$

**And** $M = .5441$

So

$x = 4(-3 + .5441)$

$x = 4(-2.4559)$

$x = -9.8236$

| $R.W$ |
|---|
| $3.5 \times 10^{-3}$ |

## Ex # 3.5

**(vi)** $\log 45$

**Solution:**

$\log 45$

$Let\ \ x = \log 45$

$x = \log 3 \times 3 \times 5$

$x = \log 3^2 \times 5$

$\log_a mn = \log_a m + \log_a n$

and $\log_a m^n = n \log_a m$

$x = 2\log 3 + \log 5$

$x = 2\log 3.00 + \log 5.00$

$x = 2(0 + .4771) + (0 + .6990)$

$x = 2(0.4771) + (0.6990)$

$x = 0.9542 + 0.6990$

$x = 1.6532$

**Q2: Express each of the following as a single logarithm.**

**(i)** $3\log 2 - 4\log 3$

**Solution:**

$3\log 2 - 4\log 3$

$As\ \ \log_a m^n = n\log_a m$

$3\log 2 - 4\log 3 = \log 2^2 - \log 3^4$

$3\log 2 - 4\log 3 = \log 8 - \log 81$

$As\ \ \log_a \frac{m}{n} = \log_a m - \log_a n$

$3\log 2 - 4\log 3 = \log \frac{8}{81}$

**(ii)** $2\log 3 + 4\log 2 - 3$

**Solution:**

$2\log 3 + 4\log 2 - 3$

$As\ \ \log_a m^n = n\log_a m$

$2\log 3 + 4\log 2 - 3 = \log 3^2 + \log 2^4 - 3(1)$

$As\ \ \log 10 = 1$

So

$2\log 3 + 4\log 2 - 3 = \log 9 + \log 16 - 3(\log 10)$

$As\ \ \log_a mn = \log_a m + \log_a n$

$2\log 3 + 4\log 2 - 3 = \log 9 \times 16 - \log 10^3$

$2\log 3 + 4\log 2 - 3 = \log 9 \times 16 - \log 1000$

$As\ \ \log_a \frac{m}{n} = \log_a m - \log_a n$

$2\log 3 + 4\log 2 - 3 = \log \frac{144}{1000}$

$2\log 3 + 4\log 2 - 3 = \log 0.144$

**(iii)** $\log 5 - 1$

**Solution:**

$\log 5 - 1$

$As\ \ \log 10 = 1$

$\log 5 - 1 = \log 5 - \log 10$

$As\ \ \log_a \frac{m}{n} = \log_a m - \log_a n$

$\log 5 - 1 = \log \frac{5}{10}$

$\log 5 - 1 = \log 0.5$

**(iv)** $\frac{1}{2}\log x - 2\log 3y + 3\log z$

**Solution:**

$\frac{1}{2}\log x - 2\log 3y + 3\log z$

$As\ \ \log_a m^n = n\log_a m$

$= \log x^{\frac{1}{2}} - \log(3y)^2 + \log z^3$

$= \log \sqrt{x} - \log 9y^2 + \log z^3$

$As\ \ \log_a \frac{m}{n} = \log_a m - \log_a n$

$And\ \ \log_a mn = \log_a m + \log_a n$

$\frac{1}{2}\log x - 2\log 3y + 3\log z = \log \frac{\sqrt{x}z^3}{9y^2}$

**Q3: Find the value of '$a$' from the following equations.**

**(i)** $\log_2 6 + \log_2 7 = \log_2 a$

**Solution:**

$\log_2 6 + \log_2 7 = \log_2 a$

$As\ \ \log_a mn = \log_a m + \log_a n$

$\log_2 6 \times 7 = \log_2 a$

$\log_2 42 = \log_2 a$

Thus

$a = 42$

**(ii)** $\log_{\sqrt{3}} a = \log_{\sqrt{3}} 5 + \log_{\sqrt{3}} 8 - \log_{\sqrt{3}} 2$

**Solution:**

$\log_{\sqrt{3}} a = \log_{\sqrt{3}} 5 + \log_{\sqrt{3}} 8 - \log_{\sqrt{3}} 2$

$As\ \log_a mn = \log_a m + \log_a n$

$As\ \log_a \frac{m}{n} = \log_a m - \log_a n$

$\log_{\sqrt{3}} a = \log_{\sqrt{3}} \frac{5 \times 8}{2}$

$\log_{\sqrt{3}} a = \log_{\sqrt{3}} \frac{40}{2}$

$\log_{\sqrt{3}} a = \log_{\sqrt{3}} 20$

Thus

$a = 20$

**(iii)** $\frac{\log_7 r}{\log_7 t} = \log_a r$

**Solution:**

$\frac{\log_7 r}{\log_7 t} = \log_a r$

$As\ \log_m n = \frac{\log_a n}{\log_a m}$

$\log_t r = \log_a r$

Thus

$a = t$

**(iv)** $\log_6 25 - \log_6 5 = \log_6 a$

**Solution:**

$\log_6 25 - \log_6 5 = \log_6 a$

$As\ \log_a \frac{m}{n} = \log_a m - \log_a n$

$\log_6 \frac{25}{5} = \log_6 a$

$\log_6 5 = \log_6 a$

Thus

$a = 5$

**Q4:** **Find $\log_2 3 . \log_3 4 . \log_4 5 . \log_5 6 . \log_6 7 . \log_7 8$**

**Solution:**

$Let\ x = \log_2 3 . \log_3 4 . \log_4 5 . \log_5 6 . \log_6 7 . \log_7 8$

$As\ \log_a m^n = n \log_a m$

So

$x = \log_2 4 . \log_4 5 . \log_5 6 . \log_6 7 . \log_7 8$

$x = \log_2 5\ .\ \log_5 6 . \log_6 7 . \log_7 8$

$x = \log_2 6\ . \log_6 7 . \log_7 8$

$x = \log_2 7\ . \log_7 8$

$x = \log_2 8$

$x = \log_2 2^3$

$x = 3 \log_2 2$

$As\ \log_a a = 1$

$x = 3(1)$

$x = 3$

# Ex # 3.6

**Page # 93**

**Q1:** **Simplify $3.81 \times 43.4$ with the help of logarithm.**

**Solution:**

**(i)** $3.81 \times 43.4$

Let $x = 3.81 \times 43.4$

Taking log on B.S

$\log x = \log 3.81 \times 43.4$

As $\log mn = \log m + \log n$

$\log x = \log 3.81 + \log 43.4$

$\log x = (0 + .5809) + (1 + .6375)$

$\log x = 0.5809 + 1.6375$

$\log x = 2.2184$

Taking anti $-$ log on B.S

Anti $-$ log $(\log x)$ = Anti $-$ log 2.2184

$x$ = Anti $-$ log 2.2184

Here

Characteristics = 2

Mantissa = .2184

So

$x = 1.654 \times 10^2$

$x = 16.54$

| |
|---|
| $\log 3.81$<br>$Ch = 0$<br>$M = .5809$ |
| $\log 43.4$<br>$Ch = 1$<br>$M = .6375$ |

| |
|---|
| $1652 + 2$<br>$= 1654$ |

**Ex # 3.6**

**(ii)** $\mathbf{73.42 \times 0.00462 \times 0.5143}$

**Solution:**

$73.42 \times 0.00462 \times 0.5143$

Let $x = 73.42 \times 0.00462 \times 0.5143$

Taking log on B.S

$\log x = 73.42 \times 0.00462 \times 0.5143$

As $\log mn = \log m + \log n$

$\log x = \log 73.42 + \log 0.00462 + \log 0.5143$

$\log x = (1 + .8658) + (-3 + .6646) + (-1 + .7113)$

$\log x = 1.8658 + (-2.3354) + (-0.2887)$

$\log x = 1.8658 - 2.3354 - 0.2887$

$\log x = -0.7583$

Add and Subtract $-1$

$\log x = -1 + 1 - 0.7583$

$\log x = -1 + .2417$

$\log x = \bar{1}\,.2417$

Taking $\text{anti} - \log \text{on B.S}$

$\text{anti} - \log\ (\log x) = \text{anti} - \log \bar{1}\,.2417$

$x = \text{anti} - \log \bar{1}\,.2417$

Here

Characteristics $= -1$

Mantissa $= .2417$

So

$x = 1.745 \times 10^{-1}$

$x = 0.1745$

| |
|---|
| $\log 73.42$<br>$Ch = 1$<br>$8657 + 1$<br>$M = .8658$ |
| $\log 0.00462$<br>$Ch = -3$<br>$M = .6646$ |
| $\log 0.5143$<br>$Ch = -1$<br>$7110 + 3$<br>$M = .7113$ |

| |
|---|
| $1742 + 3$<br>$= 1745$ |

**(iii)** $\mathbf{\dfrac{784.6 \times 0.0431}{28.23}}$

**Solution:**

$\dfrac{784.6 \times 0.0431}{28.23}$

$Let\ x = \dfrac{784.6 \times 0.0431}{28.23}$

**Taking log on B.S**

$\log x = \log \dfrac{784.6 \times 0.0431}{28.23}$

$\boldsymbol{As\ \ \log \frac{m}{n} = \log m - \log n}$

$\log x = \log 784.6 \times 0.0431 - \log 28.23$

**As** $\boldsymbol{\log mn = \log m + \log n}$

$\log x = \log 784.6 + \log 0.0431 - \log 28.23$

**Ex # 3.6**

$\log x = (2 + .8946) + (-2 + .6345) + (1 + .4507)$

$\log x = 2.8946 + (-1.3655) + (1.4507)$

$\log x = 2.8946 - 1.3655 - 1.4507$

$\log x = 0\,.0784$

Taking anti – log on B.S

$\text{anti} - \log\ (\log x) = \text{anti} - \log 0\,.0784$

$x = \text{anti} - \log 0\,.0784$

Here

Characteristics = 0

Mantissa = .0784

So

$x = 1.198 \times 10^0$

$x = 1.198$

| |
|---|
| $\log 784.6$ <br> $Ch = 2$ <br> $8943 + 3$ <br> $M = .8946$ |
| $\log 0.0431$ <br> $Ch = -2$ <br> $M = .6345$ |
| $\log 28.23$ <br> $Ch = 1$ <br> $4502 + 5$ <br> $M = .4507$ |

| |
|---|
| $1197 + 1$ <br> $= 1198$ |

**(iv)** $\dfrac{\mathbf{0.4932 \times 653.7}}{\mathbf{0.07213 \times 8456}}$

**Solution:**

$$\frac{0.4932 \times 653.7}{0.07213 \times 8456}$$

$$Let\ x = \frac{0.4932 \times 653.7}{0.07213 \times 8456}$$

Taking log on B.S

$$\log x = \log \frac{0.4932 \times 653.7}{0.07213 \times 8456}$$

$$\boldsymbol{As\ \ \log \frac{m}{n} = \log m - \log n}$$

$\log x = \log(0.4932 \times 653.7) - \log(0.07213 \times 8456)$

**As** $\boldsymbol{\log mn = \log m + \log n}$

$\log x = \log 0.4932 + \log 653.7 - (\log 0.07213 + \log 8456)$

$\log x = \log 0.4932 + \log 653.7 - \log 0.07213 - \log 8456$

$\log x = (-1 + .6930) + (2 + .8154) - (-2 + .8581) - (3 + .9271)$

$\log x = (-1 + .6930) + (2 + .8154) - (-2 + .8581) - (3 + .9271)$

$\log x = (-0.3070) + (2.8154) - (-1.1419) - (3.9271)$

$\log x = -0.3070 + 2.8154 + 1.1419 - 3.9271$

$\log x = -0.2768$

| |
|---|
| $\log 0.4932$ <br> $Ch = -1$ <br> $6928 + 2$ <br> $M = .6930$ |
| $\log 653.7$ <br> $Ch = 2$ <br> $8149 + 5$ <br> $M = .8154$ |
| $\log 0.07213$ <br> $Ch = -2$ <br> $8579 + 2$ <br> $M = .8581$ |
| $\log 8456$ <br> $Ch = 3$ <br> $9269 + 3$ <br> $M = .9272$ |

**Ex # 3.6**

Add and Subtract $-1$

$\log x = -1 + 1 - 0.2768$

$\log x = -1 + .7232$

$\log x = \bar{1}\,.7232$

Taking anti – log on B. S

$\text{anti} - \log\ (\log x) = \text{anti} - \log \bar{1}\,.7232$

$x = \text{anti} - \log \bar{1}\,..7232$

| |
|---|
| $5284 + 2$ <br> $= 5286$ |

Here

Characteristics $= -1$

Mantissa $= .7232$

So

$x = 5.286 \times 10^{-1}$

$x = 0.5286$

**(v)** $\dfrac{\mathbf{(78.41)^3\sqrt{142.3}}}{\mathbf{\sqrt[4]{0.1562}}}$

**Solution:**

$$\frac{(78.41)^3\sqrt{142.3}}{\sqrt[4]{0.1562}}$$

$$Let\ x = \frac{(78.41)^3\sqrt{142.3}}{\sqrt[4]{0.1562}}$$



Taking log on B.S

$$\log x = \log \frac{(78.41)^3\sqrt{142.3}}{\sqrt[4]{0.1562}}$$

$$\boldsymbol{As\ \ \log\frac{m}{n} = \log m - \log n}$$

$$\log x = \log(78.41)^3\sqrt{142.3} - \log \sqrt[4]{0.1562}$$

$$\textbf{As } \boldsymbol{\log mn = \log m + \log n}$$

$$\log x = \log(78.41)^3 + \log\sqrt{142.3} - \log\sqrt[4]{0.1562}$$

$$\log x = \log(78.41)^3 + \log(142.3)^{\frac{1}{2}} - \log(0.1562)^{\frac{1}{4}}$$

$$\log x = 3\log 78.41 + \frac{1}{2}\log 142.3 - \frac{1}{4}\log 0.1562$$

$$\log x = 3\log(78.41) + \frac{1}{2}\log(142.3) - \frac{1}{4}\log(0.1562)$$

$$\log x = 3\,(1 + .8944) + \frac{1}{2}(\,2 + .1532) - \frac{1}{4}(-1 + .1937)$$

| |
|---|
| $\log 78.41$ <br> $Ch = 1$ <br> $8943 + 1$ <br> $M = .8944$ |
| $\log 142.3$ <br> $Ch = 2$ <br> $1523 + 9$ <br> $M = .1523$ |
| $\log 0.1562$ <br> $Ch = -1$ <br> $1931 + 6$ <br> $M = .1937$ |

$$\log x = 3\,(1.8944) + \frac{1}{2}(\,2.1532) - \frac{1}{4}(-0.8063)$$

**Ex # 3.6**

$\log x = 5.6832 + 1.0766 + 0.2016$
$\log x = 6.9614$
Taking anti – log on B.S
$\text{anti} - \log\ (\log x) = \text{anti} - \log 6.9614$
$x = \text{anti} - \log 6.9614$
Here
Characteristics = 6
Mantissa = .9614

$\boxed{\begin{array}{l} 9141 + 8 \\ = 9149 \end{array}}$

So
$x = 9.149 \times 10^6$
$x = 9149000$

**Q2: Find the following if $\log 2 = 0.3010$, $\log 3 = 0.4771$, $\log 5 = 0.6990$, $\log 7 = 0.8451$**

**(i) $\log 105$**

**Solution:**

$\log 105$
$\log 105 = \log 3 \times 5 \times 7$
As $\log mn = \log m + \log n$
$\log 105 = \log 3 + \log 5 + \log 7$
$\log 105 = 0.4771 + 0.6990 + 0.8451$
$\log 105 = 2.0211$

**(ii) $\log 108$**

$\log 108$
$\log 108 = \log 2 \times 2 \times 3 \times 3 \times 3$
$\log 108 = \log 2^2 \times 3^3$
As $\log mn = \log m + \log n$
$\log 108 = \log 2^2 + \log 3^3$
*As* $\log_a m^n = n \log_a m$
$\log 108 = 2 \log 2 + 3 \log 3$
$\log 108 = 2(0.3010) + 3(0.4771)$
$\log 108 = 0.6020 + 1.4313$
$\log 108 = 2.0333$

**(iii) $\log \sqrt[3]{72}$**

**Solution:**

$\log \sqrt[3]{72}$
$\log \sqrt[3]{72} = \log (72)^{\frac{1}{3}}$

**Review Ex # 3**

*As* $\log_a m^n = n \log_a m$
$\log \sqrt[3]{72} = \frac{1}{3} \log 72$
$\log \sqrt[3]{72} = \frac{1}{3}(\log 2 \times 2 \times 2 \times 3 \times 3)$
$\log \sqrt[3]{72} = \frac{1}{3}(\log 2^3 \times 3^2)$
As $\log mn = \log m + \log n$
$\log \sqrt[3]{72} = \frac{1}{3}(\log 2^3 + \log 3^2)$
$\log \sqrt[3]{72} = \frac{1}{3}(3 \log 2 + 2 \log 3)$
$\log \sqrt[3]{72} = \frac{1}{3}[3(0.3010) + 2(0.4771)]$
$\log \sqrt[3]{72} = \frac{1}{3}[0.9030 + 0.9542]$
$\log \sqrt[3]{72} = \frac{1}{3}[1.8572]$
$\log \sqrt[3]{72} = 0.6191$

**(iv) $\log 2.4$**

**Solution:**

$\log 2.4$
$\log 2.4 = \log \frac{24}{10}$
*As* $\log_a \frac{m}{n} = \log_a m - \log_a n$
$\log 2.4 = \log 24 - \log 10$
$\log 2.4 = \log 2 \times 2 \times 2 \times 3 - \log 10$
$\log 2.4 = \log 2^3 \times 3 - \log 10$
As $\log mn = \log m + \log n$
$\log 2.4 = \log 2^3 + \log 3 - \log 10$
*As* $\log_a m^n = n \log_a m$
$\log 2.4 = 3 \log 2 + \log 3 - \log 10$
$\log 2.4 = 3(0.3010) + 0.4771 - \log 10$
$\log 2.4 = 0.9030 + 0.4771 - 1 \because \log 10 = 1$
$\log 2.4 = 1.3801 - 1$
$\log 2.4 = 0.3801$

**Ex # 3.6**

**(v)** $\log 0.0081$

**Solution:**

$\log 0.0081$

$$\log 0.0081 = \log \frac{81}{10000}$$

$$\log 0.0081 = \log \frac{3^4}{10^4}$$

$$\log 0.0081 = \log \left(\frac{3}{10}\right)^4$$

***As*** $\log_a m^n = n \log_a m$

$$\log 0.0081 = 4 \log \frac{3}{10}$$

***As*** $\log_a \frac{m}{n} = \log_a m - \log_a n$

$$\log 0.0081 = 4(\log 3 - \log 10)$$

$$\log 0.0081 = 4(0.4771 - 1) \quad \because \log 10 = 1$$

$$\log 0.0081 = 4(-0.5229)$$

$$\log 0.0081 = -2.0916$$

## REVIEW EXERCISE # 3

**Page # 95**

**Q2: Write 9473.2 in scientific notation.**

9473.2

In scientific notation:

$$9.4732 \times 10^3$$

**Q3: Write $5.4 \times 10^6$ in standard notation.**

$$5.4 \times 10^6$$

In standard form:

5400000

**Q4: Write in logarithm form: $3^{-3} = \frac{1}{27}$**

$$3^{-3} = \frac{1}{27}$$

**In logarithm form:**

$$\log_3 \frac{1}{27} = -3$$

**Review Ex # 3**

**Q5: Write in exponential form: $\log_5 1 = 0$**

$$\log_5 1 = 0$$

**In exponential form:**

$$5^0 = 1$$

**Q6: Solve for $x$: $\log_4 16 = x$**

$$\log_4 16 = x$$

**In exponential form:**

$$4^x = 16$$

$$4^x = 4^2$$

So

$$x = 2$$

**Q7: Find the characteristic of the common logarithm 0.0083.**

0.0083

**In scientific notation:**

$$8.3 \times 10^{-3}$$

So Characteristics $-3$

**Q8: Find $\log 12.4$**

$$\log 12.4$$

In Scientific form:

$$1.24 \times 10^1$$

Thus Characteristics $= 1$

To find Mantissa, using Log Table:

Mantissa $= .0934$

Hence $\log 12.4 = 0.0934$

**Q9: Find the value of $'a'$,**

$\log_{\sqrt{5}} 3a = \log_{\sqrt{5}} 9 + \log_{\sqrt{5}} 2 - \log_{\sqrt{5}} 3$

**Solution:**

$$\log_{\sqrt{5}} 3a = \log_{\sqrt{5}} 9 + \log_{\sqrt{5}} 2 - \log_{\sqrt{5}} 3$$

***As*** $\log_a mn = \log_a m + \log_a n$

***As*** $\log_a \frac{m}{n} = \log_a m - \log_a n$

$$\log_{\sqrt{5}} 3a = \log_{\sqrt{5}} \frac{9 \times 2}{3}$$

$$\log_{\sqrt{5}} 3a = \log_{\sqrt{5}} 3 \times 2$$

$$\log_{\sqrt{5}} 3a = \log_{\sqrt{5}} 6$$

Thus $3a = 6$

$$a = \frac{6}{2}$$

$$a = 3$$

**Q10** $\dfrac{(63.28)^3(0.00843)^2(0.4623)}{(412.3)(2.184)^5}$

**Solution:**

$$\frac{(63.28)^3(0.00843)^2(0.4623)}{(412.3)(2.184)^5}$$

$$Let\ x=\frac{(63.28)^3(0.00843)^2(0.4623)}{(412.3)(2.184)^5}$$

Taking log on B.S

$$\log x=\log\frac{(63.28)^3(0.00843)^2(0.4623)}{(412.3)(2.184)^5}$$

$$As\ \ \log\frac{m}{n}=\log m-\log n$$

$$\log x=\log((63.28)^3(0.00843)^2(0.4623))-\log((412.3)(2.184)^5)$$

As $\log mn=\log m+\log n$

$$\log x=\log(63.28)^3+\log(0.00843)^2+\log 0.4623-(\log 412.3+\log(2.184)^5)$$

$$\log x=3\log 63.28+2\log 0.00843+\log 0.4623-(\log 412.3+5\log 2.184)$$

$$\log x=3\log 63.28+2\log 0.00843+\log 0.4623-\log 412.3-5\log 2.184$$

$$\log x=3(1+.8012)+2(-3+.9258)+(-1+.6649)-(2+.6152)-5(0+.3393)$$

$$\log x=3(1.8012)+2(-2.0742)+(-0.3351)-(2.6152)-5(0.3393)$$

$$\log x=5.4036-4.1484-0.3351-2.6152-1.6965$$

$$\log x=-3.3916$$

Add and Subtract $-4$

$\log x=-4+4-3.3916$

$\log x=-4+.6084$

$\log x=\overline{4}.6084$

Taking anti $-$ log on B. S

$\text{anti}-\log\ (\log x)=\text{anti}-\log\overline{4}.6084$

$x=\text{anti}-\log\overline{4}.6084$

Here

Characteristics $=-4$

Mantissa $=.6084$

So

$x=4.059\times10^{-4}$

$x=0.000405$

| |
|---|
| $\log 63.28$<br>$Ch=1$<br>$8007+5$<br>$M=.8012$ |
| $\log 0.00843$<br>$Ch=-3$<br>$M=.9258$ |
| $\log 0.4623$<br>$Ch=-1$<br>$6646+3$<br>$M=.6649$ |
| $\log 412.3$<br>$Ch=2$<br>$6149+3$<br>$M=.6152$ |
| $\log 2.184$<br>$Ch=0$<br>$3385+8$<br>$M=.3393$ |

| |
|---|
| $4055+4$<br>$=4059$ |

Tehkals.com
Learn & Teach

# MATHEMATICS

## Class 9th (KPK)

NAME: ______________________________

F.NAME: _____________________________

CLASS:_____________ SECTION: _________

ROLL #: ______ SUBJECT: ______________

ADDRESS: ________________________________________

_______________________________________________

SCHOOL: ________________________________________

# Chapter # 4

# UNIT # 4

## ALGEBRAIC EXPRESSIONS & ALGEBRAIC FORMULAS

### Ex # 4.1

**Algebraic Expressions**

When variables and constants are connected by algebraic operations like addition, subtraction, multiplication, division, root extraction & rising integral or fractional powers is called algebraic expressions.

**Variable:**

A quantity that value may change within the context of problem. It is unknown value.

Normally, we use English letters for variables

**Example:**

a, d, e, x, y, z

**Constant:**

A quantity that value doesn't change. It is a fixed value.

**Example:**

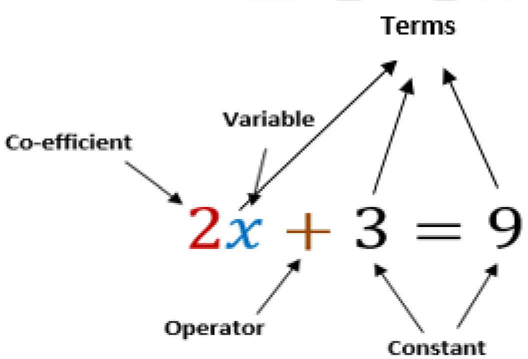
4, 6, 267, 983384

**Constant**

جس کی value تبدیل نہیں ہوتی یعنی 1,2,3,9,22

**Variable**

جس کی value تبدیل ہوتی یعنی a,b,c,x,y,z




**For Addition and Subtraction and other important terminologies**

**Visit this video:**

https://youtu.be/4jFH9OMmjXI

**Polynomial**

The algebraic expression in which powers of variables are whole numbers is called polynomial.

**Rational Expression:**

An expression of form of $\frac{p(x)}{q(x)}$ where $p(x)$ & $q(x)$ are polynomials and $q(x) \neq 0$.

**Example:**

$$\frac{x^2 - 6x + 1}{x + 9}$$

$$\frac{4x^2 + 10x + 11}{5}$$

**Note:**

Every polynomial $p(x)$ is a rational expression but every rational expression need not to be a polynomial.

**Irrational Expression:**

An expression which cannot be written in the form of $\frac{p(x)}{q(x)}$

**Term**

Different parts of an algebraic expression joined by the operations of addition and subtraction are called term.

**Example**

$3x^3 + 5\sqrt{x} - 7$. The terms are $\mathbf{3x^3}$, $\mathbf{5\sqrt{x}}$, $\mathbf{-7}$

**Rules to express a rational expression in its lowest term**

$$Let\ \frac{p(x)}{q(x)}$$

**Step 1:** Factorize both the polynomial in the numerator and denominator.

**Step 2:** cancel the common factors between them.

## Example # 9

**Page # 105**

# Chapter # 4

## Ex # 4.1

**Page # 106**

**Q1: Which of the following expressions are polynomials?**

**(i)** $\boldsymbol{1-5y+8y^2+6y^3}$

**Ans:** Polynomial and also Rational

**(ii)** $\boldsymbol{\frac{5}{x^2}+\frac{3}{4x+1}}$

**Ans:** Non-Polynomial but Rational

**(iii)** $\boldsymbol{\frac{\sqrt{x}}{6x-1}}$

**Ans:** Non-Polynomial but Irrational

**Q2: Which of the following rational expressions are in their lowest terms?**

**(i)** $\boldsymbol{\frac{5y^2-5}{y-1}}$

**Solution:**

$$\frac{5y^2--5}{y-1}$$

$$\frac{5y^2-5}{y-1}=\frac{5(y^2-1)}{y-1}$$

$$\frac{5y^2-5}{y-1}=\frac{5(y+1)(y-1)}{y-1}$$

$$\frac{5y^2-5}{y-1}=5(y+1)$$

So it is **Not** in Lowest Term:

**(ii)** $\boldsymbol{\frac{x^2-9}{x-2}}$

**Solution:**

$$\frac{x^2-9}{x-2}$$

$$\frac{x^2-9}{x-2}=\frac{(x+3)(x-3)}{x-2}$$

We can't solve it more

So it is in Lowest Term

**Ex # 4.1**

**(iii)** $\boldsymbol{\frac{x+y}{x^2-y^2}}$

**Solution:**

$$\frac{x+y}{x^2-y^2}$$

$$\frac{x+y}{x^2-y^2}=\frac{x+y}{(x+y)(x-y)}$$

$$\frac{x+y}{x^2-y^2}=\frac{1}{x-y}$$

So it is **Not** in Lowest Term:

**Q3: Reduce the following rational expression to their lowest term:**

**(i)** $\boldsymbol{\frac{x-5}{x^2-5x}}$

**Solution:**

$$\frac{x-5}{x^2-5x}$$

$$\frac{x-5}{x^2-5x}=\frac{x-5}{x(x-5)}$$

$$\frac{x-5}{x^2-5x}=\frac{1}{x}$$

**(ii)** $\boldsymbol{\frac{t^3(t-3)}{(t-3)(t+5)}}$

**Solution:**

$$\frac{t^3(t-3)}{(t-3)(t+5)}$$

$$\frac{t^3(t-3)}{(t-3)(t+5)}=\frac{t^3}{(t+5)}$$

**(iii)** $\boldsymbol{\frac{x^4+\frac{1}{x^4}}{x^2-\frac{1}{x^2}}}$

**Solution:**

$$\frac{x^4+\frac{1}{x^4}}{x^2-\frac{1}{x^2}}$$

**Ans:** It cannot be reduced further

# Chapter # 4

**Ex # 4.1**

**(iv)** $\boldsymbol{\frac{2a+6}{a^2-9}}$

**Solution:**

$\frac{2a+6}{a^2-9}$

$\frac{2a+6}{a^2-9} = \frac{2(a+3)}{(a+3)(a-3)}$

$\frac{2a+6}{a^2-9} = \frac{2}{(a-3)}$

**Q4: Add the following rational expressions:**

**(i)** $\boldsymbol{4x^2-5x-10, \quad 2x^2+5x+10}$

**Solution:**

$4x^2-5x-10, \quad 2x^2+5x+10$

Now

$(4x^2-5x-10)+(2x^2+5x+10)$

$= 4x^2-5x-10+2x^2+5x+10$

Write the like term

$= 4x^2+2x^2-5x+5x-10+10$

$= 6x^2$

**(ii)** $\boldsymbol{\frac{y+9}{y^2+3}, \quad \frac{-7y+7}{y^2+3}}$

**Solution:**

$\frac{y+9}{y^2+3}, \quad \frac{-7y+7}{y^2+3}$

$= \frac{y+9}{y^2+3} + \frac{-7y+7}{y^2+3}$

$= \frac{(y+9)+(-7y+7)}{y^2+3}$

$= \frac{y+9-7y+7}{y^2+3}$

$= \frac{y-7y+9+7}{y^2+3}$

$= \frac{-6y+16}{y^2+3}$

**Ex # 4.1**

**(iii)** $\boldsymbol{\frac{y}{y+4}, \quad \frac{2y}{y-4}}$

**Solution:**

$\frac{y}{y+4}, \quad \frac{2y}{y-4}$

$= \frac{y}{y+4} + \frac{2y}{y-4}$

$= \frac{y(y-4)+2y(y+4)}{(y+4)(y-4)}$

$= \frac{y^2-4y+2y^2+8y}{(y+4)(y-4)}$

$= \frac{y^2+2y^2-4y++8y}{x^2-4^2}$

$= \frac{3y^2+4y}{x^2-16}$

**(iv)** $\boldsymbol{\frac{t}{t^2-25}, \quad \frac{3t}{t+5}}$

**Solution:**

$\frac{t}{t^2-25}, \quad \frac{3t}{t+5}$

$\frac{t}{t^2-25} + \frac{3t}{t+5}$

$\frac{t}{(t+5)(t-5)} + \frac{3t}{t+5}$

$\frac{t+3t(t-5)}{(t+5)(t-5)}$

$\frac{t+3t^2-15t}{t^2-5^2}$

$\frac{3t^2+t-15t}{t^2-25}$

$\frac{3t^2-14t}{t^2-25}$

# Chapter # 4

Ex # 4.1

**Q5: Subtract the first expression from the second in the following.**

**(i)** $\boldsymbol{y^2+4y-15\,,\quad 8y^2+2}$

**Solution:**

$y^2+4y-15\,,\quad 8y^2+2$

$=(8y^2+2)-(y^2+4y-15)$

$=8y^2+2-y^2-4y+15$

$=8y^2-y^2-4y+2+15$

$=7y^2-4y+17$

**(ii)** $\boldsymbol{\dfrac{8x^2-7}{x^2+1}\,,\quad \dfrac{8x^2+7}{x^2+1}}$

**Solution:**

$\dfrac{8x^2-7}{x^2+1}\,,\quad \dfrac{8x^2+7}{x^2+1}$

$=\dfrac{8x^2+7}{x^2+1}-\dfrac{8x^2-7}{x^2+1}$

$=\dfrac{(8x^2+7)-(8x^2-7)}{x^2+1}$

$=\dfrac{8x^2+7-8x^2+7}{x^2+1}$

$=\dfrac{8x^2-8x^2+7+7}{x^2+1}$

$=\dfrac{14}{x^2+1}$

**(iii)** $\boldsymbol{\dfrac{1}{a-3}\,,\quad \dfrac{2a}{a^2-9}}$

**Solution:**

$\dfrac{1}{a-3}\,,\quad \dfrac{2a}{a^2-9}$

$=\dfrac{2a}{a^2-9}-\dfrac{1}{a-3}$

$=\dfrac{2a}{(a+3)(a-3)}-\dfrac{1}{a-3}$

$=\dfrac{2a-1(a+3)}{(a+3)(a-3)}$

$=\dfrac{2a-a-3}{(a+3)(a-3)}$

Ex # 4.1

$=\dfrac{a-3}{(a+3)(a-3)}$

$=\dfrac{1}{(a+3)}$

**(iv)** $\boldsymbol{\dfrac{x}{3x-6}\,,\quad \dfrac{x+2}{x-2}}$

**Solution:**

$\dfrac{x}{3x-6}\,,\quad \dfrac{x+2}{x-2}$

$=\dfrac{x+2}{x-2}-\dfrac{x}{3x-6}$

$=\dfrac{x+2}{x-2}-\dfrac{x}{3(x-2)}$

$=\dfrac{3(x+2)-x}{3(x-2)}$

$=\dfrac{3x+6-x}{3(x-2)}$

$=\dfrac{3x-x+6}{3(x-2)}$

$=\dfrac{2x+6}{3(x-2)}$

$=\dfrac{2(x+3)}{3(x-2)}$

**Q6: Simplify the following.**

**(i)** $\boldsymbol{\dfrac{2x}{6x-9}\cdot\dfrac{4x-6}{x^2+x}}$

**Solution:**

$\dfrac{2x}{6x-9}\cdot\dfrac{4x-6}{x^2+x}$

$=\dfrac{2x}{3(2x-3)}\cdot\dfrac{2(2x-3)}{x(x+1)}$

$=\dfrac{2}{3}\cdot\dfrac{2}{(x+1)}$

$=\dfrac{4}{3(x+1)}$

# Chapter # 4

**Ex # 4.1**

**(ii)** $\frac{x+4}{3-x} \cdot \frac{x^2-9}{x^2-16}$

**Solution:**

$\frac{x+4}{3-x} \cdot \frac{x^2-9}{x^2-16}$

$= \frac{x+4}{-x+3} \cdot \frac{x^2-3^2}{x^2-4^2}$

$= \frac{x+4}{-(x-3)} \cdot \frac{(x+3)(x-3)}{(x+4)(x-4)}$

$= \frac{1}{-1} \cdot \frac{(x+3)}{(x-4)}$

$= \frac{1(x+3)}{-1(x-4)}$

$= \frac{x+3}{-x+4}$

$= \frac{x+3}{4-x}$

**(iii)** $\frac{3x-15}{2x+6} \cdot \frac{x^2-9}{x^2-25}$

**Solution:**

$\frac{3x-15}{2x+6} \cdot \frac{x^2-9}{x^2-25}$

$= \frac{3(x-5)}{2(x+3)} \cdot \frac{(x+3)(x-3)}{(x+5)(x-5)}$

$= \frac{3}{2} \cdot \frac{(x-3)}{(x-5)}$

$= \frac{3(x-3)}{2(x-5)}$

**Q7:** **Simplify the following.**

**(i)** $\frac{2y-10}{3y} \div (y-5)$

**Solution:**

$\frac{2y-10}{3y} \div (y-5)$

$= \frac{2(y-5)}{3y} \times \frac{1}{y-5}$

$= \frac{2}{3y}$

**Ex # 4.1**

**(ii)** $\frac{p}{q} \div \frac{r}{q} \cdot \frac{p}{q}$

**Solution:**

$\frac{p}{q} \div \frac{r}{q} \cdot \frac{p}{q}$

$= \frac{p}{q} \cdot \frac{q}{r} \cdot \frac{p}{q}$

$= \frac{p}{q} \cdot \frac{1}{r} \cdot \frac{p}{1}$

$= \frac{p^2}{qr}$

**(iii)** $\frac{a^2-9}{(a-6)(a+4)} \div \frac{a-3}{a-6}$

**Solution:**

$\frac{a^2-9}{(a-6)(a+4)} \div \frac{a-3}{a-6}$

$= \frac{(a+3)(a-3)}{(a-6)(a+4)} \times \frac{a-6}{a-3}$

$= \frac{(a+3)}{(a+4)}$

$= \frac{a+3}{a+4}$

## Ex # 4.2

**Page # 108**

**Q1:** **Evaluate the following when $a = 3$, $b = -1$, $c = 2$.**

**(i)** $5a - 10$

**Solution:**

$5a - 10$

$5a - 10 = 5(3) - 10$

$5a - 10 = 15 - 10$

$5a - 10 = 5$

# Chapter # 4

## Ex # 4.2

**(ii)** $\mathbf{3b + 5c}$

**Solution:**

$3b + 5c$

$3b + 5c = 3(-1) + 5(2)$

$3b + 5c = -3 + 10$

$3b + 5c = 7$

**(iii)** $\mathbf{2a - 3b + 2c}$

**Solution:**

$2a - 3b + 2c$

$2a - 3b + 2c = 2(3) - 3(-1) + 2(2)$

$2a - 3b + 2c = 6 + 3 + 4$

$2a - 3b + 2c = 13$

**Q2: Evaluate the following for $x = -5$ and $y = 2$.**

**(i)** $\mathbf{7 - 3xy}$

**Solution:**

$7 - 3xy$

$7 - 3xy = 7 - 3(-5)(2)$

$7 - 3xy = 7 - 3(-10)$

$7 - 3xy = 7 + 30$

$7 - 3xy = 37$

**(ii)** $\mathbf{x^2 + xy + y^2}$

**Solution:**

$x^2 + xy + y^2$

$x^2 + xy + y^2 = (-5)^2 + (-5)(2) + (2)^2$

$x^2 + xy + y^2 = 25 + (-10) + 4$

$x^2 + xy + y^2 = 25 - 10 + 4$

$x^2 + xy + y^2 = 15 + 4$

$x^2 + xy + y^2 = 19$

**(iii)** $\mathbf{(3x)^2 - (4y)^2}$

**Solution:**

$(3x)^2 - (4y)^2$

$(3x)^2 - (4y)^2 = [3(-5)]^2 - [4(2)]^2$

$(3x)^2 - (4y)^2 = [-15]^2 - [8]^2$

$(3x)^2 - (4y)^2 = 225 - 64$

$(3x)^2 - (4y)^2 = 161$

## Ex # 4.2

**Q3: Evaluate the following when $k = -2$ , $l = 3$ , $m = 4$.**

**(i)** $\mathbf{k^2(2l - 3m)}$

**Solution:**

$k^2(2l - 3m)$

$k^2(2l - 3m) = (-2)^2[2(3) - 3(4)]$

$k^2(2l - 3m) = 4(6 - 12)$

$k^2(2l - 3m) = 4(-6)$

$k^2(2l - 3m) = -24$

**(ii)** $\mathbf{5m\sqrt{k^2 + l^2}}$

**Solution:**

$5m\sqrt{k^2 + l^2}$

$5m\sqrt{k^2 + l^2} = 5(4)\sqrt{(-2)^2 + (3)^2}$

$5m\sqrt{k^2 + l^2} = 20\sqrt{4 + 9}$

$5m\sqrt{k^2 + l^2} = 20\sqrt{13}$

**(iii)** $\mathbf{\dfrac{k + l + m}{k^2 + l^2 + m^2}}$

**Solution:**

$\dfrac{k + l + m}{k^2 + l^2 + m^2}$

Put the values

$\dfrac{k + l + m}{k^2 + l^2 + m^2} = \dfrac{(-2) + (3) + (4)}{(-2)^2 + (3)^2 + (4)^2}$

$\dfrac{k + l + m}{k^2 + l^2 + m^2} = \dfrac{-2 + 3 + 4}{4 + 9 + 16}$

$\dfrac{k + l + m}{k^2 + l^2 + m^2} = \dfrac{1 + 4}{13 + 16}$

$\dfrac{k + l + m}{k^2 + l^2 + m^2} = \dfrac{5}{29}$

# Chapter # 4

### Ex # 4.2

**Q4:** $\boldsymbol{Evaluate\ \frac{a+1}{4a^2+1}\ when\ a=\frac{1}{2}\ \text{and}\ a=-\frac{1}{2}.}$

**Solution:**

$\boldsymbol{For\ a=-\frac{1}{2}}$

$$\frac{a+1}{4a^2+1}$$

$$\frac{a+1}{4a^2+1}=\frac{\frac{1}{2}+1}{4\left(\frac{1}{2}\right)^2+1}$$

$$\frac{a+1}{4a^2+1}=\frac{\frac{1+2}{2}}{4\left(\frac{1}{4}\right)+1}$$

$$\frac{a+1}{4a^2+1}=\frac{\frac{3}{2}}{1+1}$$

$$\frac{a+1}{4a^2+1}=\frac{\frac{3}{2}}{2}$$

$$\frac{a+1}{4a^2+1}=\frac{3}{2}\div 2$$

$$\frac{a+1}{4a^2+1}=\frac{3}{2}\times\frac{1}{2}$$

$$\frac{a+1}{4a^2+1}=\frac{3}{4}$$

$\boldsymbol{For\ a=-\frac{1}{2}}$

$$\frac{a+1}{4a^2+1}$$

$$\frac{a+1}{4a^2+1}=\frac{-\frac{1}{2}+1}{4\left(-\frac{1}{2}\right)^2+1}$$

$$\frac{a+1}{4a^2+1}=\frac{\frac{-1+2}{2}}{4\left(\frac{1}{4}\right)+1}$$

$$\frac{a+1}{4a^2+1}=\frac{\frac{1}{2}}{1+1}$$

### Ex # 4.2

$$\frac{a+1}{4a^2+1}=\frac{\frac{1}{2}}{2}$$

$$\frac{a+1}{4a^2+1}=\frac{1}{2}\div 2$$

$$\frac{a+1}{4a^2+1}=\frac{1}{2}\times\frac{1}{2}$$

$$\frac{a+1}{4a^2+1}=\frac{1}{4}$$

**Q5:** $\boldsymbol{If\ a=9,\quad b=12, c=15\ and\ s=\frac{a+b+c}{2}.}$ $\boldsymbol{Find\ the\ \ value\ of\ \sqrt{s(s-a)(s-b)(s-c)}}$

**Solution:**

<u>Given:</u>

$$a=9\,,b=12,c=15\ and\ s=\frac{a+b+c}{2}$$

**To Find:**

$$\sqrt{s(s-a)(s-b)(s-c)}=?$$

<u>First we find:</u>

$$s=\frac{a+b+c}{2}$$

Put the values:

$$s=\frac{a+b+c}{2}$$

$$s=\frac{9+12+15}{2}$$

$$s=\frac{36}{2}$$

$$s=18$$

Now

$$\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{18(18-9)(18-12)(18-15)}$$

$$\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{18(9)(6)(3)}$$

$$\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{2\times9\times9\times2\times3\times3}$$

$$\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{9\times9\times2\times2\times3\times3}$$

$$\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{9^2\times2^2\times3^2}$$

$$\sqrt{s(s-a)(s-b)(s-c)}=9\times2\times3$$

$$\sqrt{s(s-a)(s-b)(s-c)}=9\times6$$

$$\sqrt{s(s-a)(s-b)(s-c)}=54$$

**Ex # 4.3**

1. $(a+b)^2 = a^2+b^2+2ab$
2. $(a-b)^2 = a^2+b^2-2ab$
3. $a^2-b^2 = (a+b)(a-b)$
4. $(a+b)^2+(a-b)^2 = 2(a^2+b^2)$ **Q2, Q3(i)**
5. $(a+b)^2-(a-b)^2 = 4ab$ **Q2, Q3(ii)**
6. $(x+y)^2+(x-y)^2 = 2(x^2+y^2)$ **Q1, Q5**
7. $(x+y)^2-(x-y)^2 = 4xy$ **Q1, Q4, Q5**
8. $(u+v)^2-(u-v)^2 = 4uv$ **Q6**

# Ex # 4.3

**Page # 110**

**Q1: Find the value of $x^2+y^2$ and $xy$, when:**

**(i)** $x+y=8,\quad x-y=3$

**Solution:**

$x+y=8,\quad x-y=3$

**To Find:**

$x^2+y^2$ =? and $xy$ =?

***$x^2+y^2$***

As we have

$(x+y)^2+(x-y)^2 = 2(x^2+y^2)$

Put the values

$(8)^2+(3)^2 = 2(x^2+y^2)$

$64+9 = 2(x^2+y^2)$

$73 = 2(x^2+y^2)$

Divide B.S by 2

$\frac{73}{2} = \frac{2(x^2+y^2)}{2}$

$\frac{73}{2} = x^2+y^2$

$x^2+y^2 = \frac{73}{2}$

***$xy$***

Also we have

$(x+y)^2-(x-y)^2 = 4xy$

Put the values

$(8)^2-(3)^2 = 4xy$

$64-9 = 4xy$

$55 = 4xy$

Divide B.S by 4

$\frac{55}{4} = \frac{4xy}{4}$

$\frac{55}{4} = xy$

$xy = \frac{55}{4}$

**Ex # 4.3**

**(ii)** $x+y=10,\quad x-y=7$

**Solution:**

$x+y=10,\quad x-y=7$

**To Find:**

$x^2+y^2$ =? And $xy$ =?

***$x^2+y^2$***

As we have

$(x+y)^2+(x-y)^2 = 2(x^2+y^2)$

Put the values

$(10)^2+(7)^2 = 2(x^2+y^2)$

$100+49 = 2(x^2+y^2)$

$149 = 2(x^2+y^2)$

Divide B.S by 2

$\frac{149}{2} = \frac{2(x^2+y^2)}{2}$

$\frac{149}{2} = x^2+y^2$

$x^2+y^2 = \frac{149}{2}$

***$xy$***

Also we have

$(x+y)^2-(x-y)^2 = 4xy$

Put the values

$(10)^2-(7)^2 = 4xy$

$100-49 = 4xy$

$51 = 4xy$

Divide B.S by 4

$\frac{51}{4} = \frac{4xy}{4}$

$\frac{51}{4} = xy$

$xy = \frac{51}{4}$

**(iii)** $x+y=11,\quad x-y=5$

**Solution:**

$x+y=11,\quad x-y=5$

**To Find:**

$x^2+y^2$ =? and $xy$ =?

***$x^2+y^2$***

As we have

$(x+y)^2+(x-y)^2 = 2(x^2+y^2)$

Put the values

$(11)^2+(5)^2 = 2(x^2+y^2)$

# Chapter # 4

**Ex # 4.3**

$121+25=2(x^2+y^2)$

$146=2(x^2+y^2)$

Divide B.S by 2

$\frac{146}{2}=\frac{2(x^2+y^2)}{2}$

$73=x^2+y^2$

$x^2+y^2=73$

***$xy$***

Also we have

$$(x+y)^2-(x-y)^2=4xy$$

Put the values

$(11)^2-(5)^2=4xy$

$121-25=4xy$

$96=4xy$

Divide B.S by 4

$\frac{96}{4}=\frac{4xy}{4}$

$24=xy$

$xy=24$

**(iv)** $x+y=7,\quad x-y=4$

**Solution:**

$x+y=7,\quad x-y=4$

**To Find:**

$x^2+y^2=?$ and $xy=?$

***$x^2+y^2$***

As we have

$$(x+y)^2+(x-y)^2=2(x^2+y^2)$$

Put the values

$(7)^2+(4)^2=2(x^2+y^2)$

$49+16=2(x^2+y^2)$

$65=2(x^2+y^2)$

Divide B.S by 2

$\frac{65}{2}=\frac{2(x^2+y^2)}{2}$

$\frac{65}{2}=x^2+y^2$

$x^2+y^2=\frac{65}{2}$

***$xy$***

Also we have

$$(x+y)^2-(x-y)^2=4xy$$

Put the values

$(7)^2-(4)^2=4xy$

**Ex # 4.3**

$49-16=4xy$

$33=4xy$

Divide B.S by 4

$\frac{33}{4}=\frac{4xy}{4}$

$\frac{33}{4}=xy$

$xy=\frac{33}{4}$

**Q2: Find the value of $a^2+b^2$ and $ab$, when**

**(i)** $a+b=7,\qquad a-b=3$

**Solution:**

$a+b=7$ and $a-b=3$

**To Find:**

$a^2+b^2=?$ and $ab=?$

***$a^2+b^2$***

As we have

$$(a+b)^2+(a-b)^2=2(a^2+b^2)$$

Put the values

$(7)^2+(3)^2=2(a^2+b^2)$

$49+9=2(a^2+b^2)$

$58=2(a^2+b^2)$

Divide B.S by 2

$\frac{58}{2}=\frac{2(a^2+b^2)}{2}$

$29=a^2+b^2$

$a^2+b^2=29$

***$ab$***

Also we have

$$(a+b)^2-(a-b)^2=4ab$$

Put the values

$(7)^2-(3)^2=4ab$

$49-9=4ab$

$40=4ab$

Divide B.S by 4

$\frac{40}{4}=\frac{4ab}{4}$

$10=ab$

$ab=10$

# Chapter # 4

## Ex # 4.3

**Q2: (ii)** **Find the value of $a^2 + b^2$ and $ab$, when $a + b = 9$, $a - b = 1$.**

**Solution:**

$a + b = 9$ and $a - b = 1$

**To Find:**

$a^2 + b^2 =?$ and $ab =?$

$\underline{a^2 + b^2}$

As we have

$$(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$$

Put the values

$(9)^2 + (1)^2 = 2(a^2 + b^2)$

$81 + 1 = 2(a^2 + b^2)$

$82 = 2(a^2 + b^2)$

Divide B.S by 2

$\frac{82}{2} = \frac{2(a^2 + b^2)}{2}$

$41 = a^2 + b^2$

$a^2 + b^2 = 41$

$\underline{ab}$

Also we have

$$(a + b)^2 - (a - b)^2 = 4ab$$

Put the values

$(9)^2 - (1)^2 = 4ab$

$81 - 1 = 4ab$

$80 = 4ab$

Divide B.S by 4

$\frac{80}{4} = \frac{4ab}{4}$

$20 = ab$

$ab = 20$

**Q3: (i)** **If $a + b = 10$, $a - b = 6$, then find the value of $a^2 + b^2$.**

**Solution:**

$a + b = 10$ and $a - b = 6$

**To Find:**

$a^2 + b^2 =?$

As we have

$$(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$$

Put the values

$(10)^2 + (6)^2 = 2(a^2 + b^2)$

$100 + 36 = 2(a^2 + b^2)$

$136 = 2(a^2 + b^2)$

## Ex # 4.3

Divide B.S by 2

$\frac{136}{2} = \frac{2(a^2 + b^2)}{2}$

$68 = a^2 + b^2$

$a^2 + b^2 = 68$

**Q3: (ii)** **If $a + b = 5$, $a - b = \sqrt{17}$, then find the value of $ab$.**

**Solution:**

$a + b = 5$ and $a - b = \sqrt{17}$

**To Find:**

$$ab =?$$

Also we have

$$(a + b)^2 - (a - b)^2 = 4ab$$

Put the values

$(5)^2 - (\sqrt{17})^2 = 4ab$

$25 - 17 = 4ab$

$8 = 4ab$

Divide B.S by 4

$\frac{8}{4} = \frac{4ab}{4}$

$2 = ab$

$ab = 2$

**Q4:** **Find the value of $4xy$ when $x + y = 17$, $x - y = 5$.**

**Solution:**

$$x + y = 17, \quad x - y = 5$$

**To find:**

$$4xy =?$$

Also we have

$$(x + y)^2 - (x - y)^2 = 4xy$$

Put the values

$(17)^2 - (5)^2 = 4xy$

$289 - 25 = 4xy$

$264 = 4xy$

OR

$4xy = 264$

# Chapter # 4

**Ex # 4.3**

**Q5:** **If $+y = 11$ *and* $x - y = 3$ ,**
**find $8xy(x^2 + y^2)$.**

**Solution:**

$x + y = 11, \quad x - y = 3$

**To Find:**

$8xy(x^2 + y^2) = ?$

As we have

$$(x + y)^2 + (x - y)^2 = 2(x^2 + y^2)$$

Put the values

$(11)^2 + (3)^2 = 2(x^2 + y^2)$

$121 + 9 = 2(x^2 + y^2)$

$130 = 2(x^2 + y^2)$

$2(x^2 + y^2) = 130 \quad --equ(i)$

Also we have

$$(x + y)^2 - (x - y)^2 = 4xy$$

Put the values

$(11)^2 - (3)^2 = 4xy$

$121 - 9 = 4xy$

$112 = 4xy$

$4xy = 112 \quad --equ(ii)$

Multiply equ (i) and (ii)

$2(x^2 + y^2) \times 4xy = 130 \times 112$

$8xy(x^2 + y^2) = 14560$

**Q6:** **If $u + v = 7$ and $uv = 12$, find $u - v$.**

**Solution:**

$u + v = 7, \; uv = 12$

**To Find:**

$u - v = ?$

As we know that

$$(u + v)^2 - (u - v)^2 = 4uv$$

Put the values

$(7)^2 - (u - v)^2 = 4(12)$

$49 - (u - v)^2 = 48$

$-(u - v)^2 = 48 - 49$

$-(u - v)^2 = -1$

$(u - v)^2 = 1$

Taking square root on B.S

$\sqrt{(u - v)^2} = \sqrt{1}$

$u - v = \pm 1$

## Ex # 4.4

1. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
   **Q1, Q2, Q3**
2. $2(x^2 + y^2 + z^2 - xy - yz - zx) =$
   $(x - y)^2 + (y - z)^2 + (z - x)^2$ **Q4, Q5**
3. $2(a^2 + b^2 + c^2 - ab - bc - ca) =$
   $(a - b)^2 + (b - c)^2 + (c - a)^2$ **Q6**

# Ex # 4.4

**Page # 112**

**Q1:** **Find the values of $a^2 + b^2 + c^2$, when**

**(i)** **$a + b + c = 5$ and $ab + bc + ca = -4$**

**Solution:**

$a + b + c = 5$ and $ab + bc + ca = -4$

To Find:

$a^2 + b^2 + c^2 = ?$

As we know that

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

Put the values

$(5)^2 = a^2 + b^2 + c^2 + 2(-4)$

$25 = a^2 + b^2 + c^2 - 8$

$25 + 8 = a^2 + b^2 + c^2$

$33 = a^2 + b^2 + c^2$

$a^2 + b^2 + c^2 = 33$

**(ii)** **$a + b + c = 5$ and $ab + bc + ca = -2$**

**Solution:**

$a + b + c = 5$ and $ab + bc + ca = -2$

To Find:

$a^2 + b^2 + c^2 = ?$

As we know that

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

Put the values

$(5)^2 = a^2 + b^2 + c^2 + 2(-2)$

$25 = a^2 + b^2 + c^2 - 4$

$25 + 4 = a^2 + b^2 + c^2$

$29 = a^2 + b^2 + c^2$

$a^2 + b^2 + c^2 = 29$

# Chapter # 4

**Ex # 4.4**

**Q2: Find the values of $a+b+c$, when**

**(i) $a^2+b^2+c^2=38$ and $ab+bc+ca=-1$**

**Solution:**

$a^2+b^2+c^2=38$ and $ab+bc+ca=-1$

To Find:

$a+b+c=?$

As we know that

$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

Put the values

$(a+b+c)^2=38+2(-1)$

$(a+b+c)^2=38-2$

$(a+b+c)^2=36$

Taking square root on B.S

$\sqrt{(a+b+c)^2}=\sqrt{36}$

$a+b+c=6$

**(ii) $a^2+b^2+c^2=10$ and $ab+bc+ca=11$**

**Solution:**

$a^2+b^2+c^2=10$ and $ab+bc+ca=11$

To Find:

$$a+b+c=?$$

As we know that

$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

Put the values

$(a+b+c)^2=10+2(11)$

$(a+b+c)^2=10+22$

$(a+b+c)^2=32$

Taking square root on B.S

$\sqrt{(a+b+c)^2}=\sqrt{32}$

$a+b+c=\sqrt{16\times 2}$

$a+b+c=\sqrt{16}\times\sqrt{2}$

$a+b+c=4\sqrt{2}$

**Q3: Find the values of $ab+bc+ca$, when**

**(i) $a^2+b^2+c^2=56$ and $a+b+c=12$**

**Solution:**

$a^2+b^2+c^2=56$ and $a+b+c=12$

To Find:

$$ab+bc+ca=?$$

As we know that

$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

**Ex # 4.4**

Put the values

$(12)^2=56+2(ab+bc+ca)$

$144=56+2(ab+bc+ca)$

Subtract 56 from B.S

$144-56=56-56+2(ab+bc+ca)$

$88=2(ab+bc+ca)$

Divide B.S by 2

$\frac{88}{2}=\frac{2(ab+bc+ca)}{2}$

$44=ab+bc+ca$

$ab+bc+ca=44$

**(ii) $a^2+b^2+c^2=12$ and $a+b+c=5$**

**Solution:**

$a^2+b^2+c^2=12$ and $a+b+c=5$

To Find:

$$ab+bc+ca=?$$

As we know that

$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

Put the values

$(5)^2=12+2(ab+bc+ca)$

$25=12+2(ab+bc+ca)$

Subtract 12 from B.S

$25-12=12-12+2(ab+bc+ca)$

$13=2(ab+bc+ca)$

Divide B.S by 2

$\frac{13}{2}=\frac{2(ab+bc+ca)}{2}$

$\frac{13}{2}=ab+bc+ca$

$ab+bc+ca=\frac{13}{2}$

# Chapter # 4

## Ex # 4.4

**Q #4** **Prove that $x^2+y^2+y^2-xy-yz-zx = \left(\frac{x-y}{\sqrt{2}}\right)^2+\left(\frac{y-z}{\sqrt{2}}\right)^2+\left(\frac{z-x}{\sqrt{2}}\right)^2$**

**Solution:**

$x^2+y^2+y^2-xy-yz-zx =$

$$\left(\frac{x-y}{\sqrt{2}}\right)^2+\left(\frac{y-z}{\sqrt{2}}\right)^2+\left(\frac{z-x}{\sqrt{2}}\right)^2$$

R.H.S

$$\left(\frac{x-y}{\sqrt{2}}\right)^2+\left(\frac{y-z}{\sqrt{2}}\right)^2+\left(\frac{z-x}{\sqrt{2}}\right)^2$$

$$=\frac{(x-y)^2}{(\sqrt{2})^2}+\frac{(y-z)^2}{(\sqrt{2})^2}+\frac{(z-x)^2}{(\sqrt{2})^2}$$

$$=\frac{x^2+y^2-2xy}{2}+\frac{y^2+z^2-2yz}{2}+\frac{z^2+x^2-2zx}{2}$$

$$=\frac{x^2+y^2-2xy+y^2+z^2-2yz+z^2+x^2-2zx}{2}$$

$$=\frac{2x^2+2y^2+2z^2-2xy-2yz-2zx}{2}$$

$$=\frac{2(x^2+y^2+z^2-xy-yz-zx)}{2}$$

$$=x^2+y^2+z^2-xy-yz-zx$$

$= \text{L.H.S}$

**Q #5** **Write $2[x^2+y^2+y^2-xy-yz-zx]$ as the sum of three squares.**

**Solution:**

$$2[x^2+y^2+y^2-xy-yz-zx]$$

$$2x^2+2y^2+2z^2-2xy-2yz-2zx$$

$$x^2+x^2+y^2+y^2+z^2+z^2-2xy-2yz-2zx$$

Re-arranging the terms

$$x^2+y^2-2xy+y^2+z^2-2yz+z^2+x^2-2zx$$

As we have

$$\boldsymbol{a^2+b^2-2ab=(a-b)^2}$$

$$(x-y)^2+(y-z)^2+(z-x)^2$$

## Ex # 4.4

**Q #6** **Find the value of**
**$a^2+b^2+c^2-ab-bc-ca$**
**when $a-b=2,\ b-c=3, c-a=4.$**

**Solution:**

Given that:

$$a-b=2,\qquad b-c=3, c-a=4$$

**To find**

$$a^2+b^2+c^2-ab-bc-ca=?$$

As we have

$$\boldsymbol{2(a^2+b^2+c^2-ab-bc-ca)=(a-b)^2+(b-c)^2+(c-a)^2}$$

Put the values

$$2(a^2+b^2+c^2-ab-bc-ca)=(2)^2+(3)^2+(4)^2$$

$$2(a^2+b^2+c^2-ab-bc-ca)=4+9+16$$

$$2(a^2+b^2+c^2-ab-bc-ca)=29$$

Divide B.S by 2

$$\frac{2(a^2+b^2+c^2-ab-bc-ca)}{2}=\frac{29}{2}$$

$$a^2+b^2+c^2-ab-bc-ca=\frac{29}{2}$$

## Ex # 4.5

1. $(a+b)^3=a^3+b^3+3ab(a+b)$ **Q#1, 7**
2. $(a-b)^3=a^3-b^3-3ab(a-b)$ **Q#2**
3. $\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3(x)\left(\frac{1}{x}\right)\left(x+\frac{1}{x}\right)$ **Q#3**
4. $\left(x-\frac{1}{x}\right)^3=x^3-\frac{1}{x^3}-3(x)\left(\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$ **Q#4**
5. $\left(3a+\frac{1}{a}\right)^3=27a^3+\frac{1}{a^3}+3(3a)\left(\frac{1}{a}\right)\left(3a+\frac{1}{a}\right)$ **Q#5**
6. $\left(x-\frac{1}{2x}\right)^3=x^3-\frac{1}{8x^3}-3(x)\left(\frac{1}{2x}\right)\left(x-\frac{1}{2x}\right)$ **Q#6**
7. $(u-v)^3=u^3-v^3-3uv(u-v)$ **Q#8**
8. $\left(a+\frac{1}{a}\right)^2=a^2+\frac{1}{a^2}+2(a)\left(\frac{1}{a}\right)$ **Q#9**
9. $\left(a^2+\frac{1}{a^2}\right)^2=a^4+\frac{1}{a^4}+2(a^2)\left(\frac{1}{a^2}\right)$ **Q#9**
10. $\left(a+\frac{1}{a}\right)^3=a^3+\frac{1}{a^3}+3(a)\left(\frac{1}{a}\right)\left(a+\frac{1}{a}\right)$

# Ex # 4.5

**Page # 115**

**Q1: Find the value of $a^3+b^3$, when**

**(i) $a+b=4$ and $ab=5$.**

**Solution:**

$$a+b=4, \qquad ab=5$$

**To Find:**

$$a^3+b^3=?$$

As we have

$$(a+b)^3=a^3+b^3+3ab(a+b)$$

Put the values

$(4)^3=a^3+b^3+3(5)(4)$

$64=a^3+b^3+60$

Subtract 60 from B.S

$64-60=a^3+b^3+60-60$

$4=a^3+b^3$

$a^3+b^3=4$

**(ii) $a+b=3$ and $ab=20$.**

**Solution:**

$a+b=3$ and $ab=20$.

**To Find:**

$$a^3+b^3=?$$

As we have

$$(a+b)^3=a^3+b^3+3ab(a+b)$$

Put the values

$(3)^3=a^3+b^3+3(3)(20)$

$27=a^3+b^3+180$

Subtract 180 from B.S

$27-180=a^3+b^3+180-180$

$-153=a^3+b^3$

$a^3+b^3=-153$

**(iii) $a+b=4$ and $ab=2$.**

**Solution:**

$a+b=4$ and $ab=2$.

**To Find:**

$$a^3+b^3=?$$

As we have

$$(a+b)^3=a^3+b^3+3ab(a+b)$$

Put the values

$(4)^3=a^3+b^3+3(2)(4)$

$64=a^3+b^3+24$



Subtract 24 from B.S

$64-24=a^3+b^3+24-24$

$40=a^3+b^3$

$a^3+b^3=40$

**Q2: Find the value of $a^3-b^3$, when**

**(i) $a-b=5$ and $ab=7$.**

**Solution:**

$a-b=5, \ ab=7$

**To Find:**

$$a^3-b^3=?$$

As we have

$$(a-b)^3=a^3-b^3-3ab(a-b)$$

Put the values

$(5)^3=a^3-b^3-3(7)(5)$

$125=a^3-b^3-105$

Add 105 on B.S

$125+105=a^3-b^3-105+105$

$230=a^3-b^3$

$a^3-b^3=230$

**(ii) $a-b=2$ and $ab=15$.**

**Solution:**

$a-b=2, \ ab=15$

**To Find:**

$$a^3-b^3=?$$

As we have

$$(a-b)^3=a^3-b^3-3ab(a-b)$$

Put the values

$(2)^3=a^3-b^3-3(15)(2)$

$8=a^3-b^3-90$

Add 90 on B.S

$8+90=a^3-b^3-90+90$

$98=a^3-b^3$

$a^3-b^3=98$

**Ex # 4.5**

**(iii)** $\boldsymbol{a - b = 7}$ **and** $\boldsymbol{ab = 6.}$

**Solution:**

$a - b = 7, \quad ab = 6$

**To Find:**

$$a^3 - b^3 = ?$$

As we have

$$(\boldsymbol{a - b})^3 = \boldsymbol{a^3 - b^3 - 3ab(a - b)}$$

Put the values

$(7)^3 = a^3 - b^3 - 3(6)(7)$

$343 = a^3 - b^3 - 126$

Add 126 on B.S

$343 + 126 = a^3 - b^3 - 126 + 126$

$469 = a^3 - b^3$

$a^3 + b^3 = 469$

**Q3: Find the value of** $\boldsymbol{x^3 + \frac{1}{x^3}}$ **, when**

**(i)** $\boldsymbol{x + \frac{1}{x} = \frac{5}{2}}$

**Solution:**

$x + \frac{1}{x} = \frac{5}{2}$

**To Find:**

$x^3 + \frac{1}{x^3} = ?$

As we have

$$\left(\boldsymbol{x + \frac{1}{x}}\right)^3 = \boldsymbol{x^3 + \frac{1}{x^3} + 3(x)\left(\frac{1}{x}\right)\left(x + \frac{1}{x}\right)}$$

Put the values

$\left(\frac{5}{2}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(\frac{5}{2}\right)$

$\frac{125}{8} = x^3 + \frac{1}{x^3} + \frac{15}{2}$

Subtract $\frac{15}{2}$ from B.S

$\frac{125}{8} - \frac{15}{2} = x^3 + \frac{1}{x^3} + \frac{15}{2} - \frac{15}{2}$

$\frac{125 - 60}{8} = x^3 + \frac{1}{x^3}$

$\frac{65}{8} = x^3 + \frac{1}{x^3}$

$x^3 + \frac{1}{x^3} = \frac{65}{8}$

**Ex # 4.5**

**(ii)** $\boldsymbol{x + \frac{1}{x} = 2}$

**Solution:**

$$x + \frac{1}{x} = 2$$

**To Find:**

$$x^3 + \frac{1}{x^3} = ?$$

As we have

$$\left(\boldsymbol{x + \frac{1}{x}}\right)^3 = \boldsymbol{x^3 + \frac{1}{x^3} + 3(x)\left(\frac{1}{x}\right)\left(x + \frac{1}{x}\right)}$$

Put the values

$(2)^3 = x^3 + \frac{1}{x^3} + 3(2)$

$8 = x^3 + \frac{1}{x^3} + 6$

Subtract 6 from B.S

$8 - 6 = x^3 + \frac{1}{x^3} + 6 - 6$

$2 = x^3 + \frac{1}{x^3}$

$x^3 + \frac{1}{x^3} = 2$

**Q3: Find the value of** $\boldsymbol{x^3 - \frac{1}{x^3}}$ **, when**

**(i)** $\boldsymbol{x - \frac{1}{x} = \frac{3}{2}}$

**Solution:**

$$x - \frac{1}{x} = \frac{3}{2}$$

**To Find:**

$$x^3 - \frac{1}{x^3} = ?$$

As we have

$$\left(\boldsymbol{x - \frac{1}{x}}\right)^3 = \boldsymbol{x^3 - \frac{1}{x^3} - 3(x)\left(\frac{1}{x}\right)\left(x - \frac{1}{x}\right)}$$

Put the values

$\left(\frac{3}{2}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(\frac{3}{2}\right)$

$\frac{27}{8} = x^3 - \frac{1}{x^3} - \frac{9}{2}$

Add $\frac{9}{2}$ on B.S

$\frac{27}{8} + \frac{9}{2} = x^3 - \frac{1}{x^3} - \frac{9}{2} + \frac{9}{2}$

# Chapter # 4

**Ex # 4.5**

$$\frac{27+36}{8}=x^3-\frac{1}{x^3}$$
$$\frac{63}{8}=x^3-\frac{1}{x^3}$$
$$x^3-\frac{1}{x^3}=\frac{63}{8}$$

**(ii)** $\boldsymbol{x-\frac{1}{x}=\frac{7}{3}}$

**Solution:**

$$x-\frac{1}{x}=\frac{7}{3}$$

**To Find:**

$$x^3-\frac{1}{x^3}=?$$

As we have

$$\left(x-\frac{1}{x}\right)^3=x^3-\frac{1}{x^3}-3(x)\left(\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$$

Put the values

$$\left(\frac{7}{3}\right)^3=x^3-\frac{1}{x^3}-3\left(\frac{7}{3}\right)$$
$$\frac{343}{27}=x^3-\frac{1}{x^3}-\frac{21}{3}$$

Add $\frac{21}{3}$ on B.S

$$\frac{343}{27}+\frac{21}{3}=x^3-\frac{1}{x^3}-\frac{21}{3}+\frac{21}{3}$$
$$\frac{343+189}{27}=x^3-\frac{1}{x^3}$$
$$\frac{532}{27}=x^3-\frac{1}{x^3}$$
$$x^3-\frac{1}{x^3}=\frac{532}{27}$$

**(iii)** $\boldsymbol{x-\frac{1}{x}=\frac{15}{4}}$

**Solution:**

$$x-\frac{1}{x}=\frac{15}{4}$$

**To Find:**

$$x^3-\frac{1}{x^3}=?$$

As we have

$$\left(x-\frac{1}{x}\right)^3=x^3-\frac{1}{x^3}-3(x)\left(\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$$

**Ex # 4.5**

Put the values

$$\left(\frac{15}{4}\right)^3=x^3-\frac{1}{x^3}-3\left(\frac{15}{4}\right)$$
$$\frac{3375}{64}=x^3-\frac{1}{x^3}-\frac{45}{4}$$

Add $\frac{45}{4}$ on B.S

$$\frac{3375}{64}+\frac{45}{4}=x^3-\frac{1}{x^3}-\frac{45}{4}+\frac{45}{4}$$
$$\frac{3375+720}{64}=x^3-\frac{1}{x^3}$$
$$\frac{4095}{64}=x^3-\frac{1}{x^3}$$
$$x^3-\frac{1}{x^3}=\frac{4095}{64}$$

**Q5:** **If $\boldsymbol{3a+\frac{1}{a}=4}$, find $\boldsymbol{27a^3+\frac{1}{a^3}}$**

**Solution:**

$$3a+\frac{1}{a}=4$$

**To Find:**

$$27a^3+\frac{1}{a^3}=?$$

As we have

$$\left(3a+\frac{1}{a}\right)^3=27a^3+\frac{1}{a^3}+3(3a)\left(\frac{1}{a}\right)\left(3a+\frac{1}{a}\right)$$

Put the values

$$(4)^3=27a^3+\frac{1}{a^3}+9(4)$$
$$64=27a^3+\frac{1}{a^3}+36$$

Subtract 36 from B.S

$$64-36=27a^3+\frac{1}{a^3}+36-36$$
$$28=27a^3+\frac{1}{a^3}$$
$$27a^3+\frac{1}{a^3}=28$$

### Ex # 4.5

**Q6:** **If $x - \frac{1}{2x} = 6$, find $x^3 - \frac{1}{8x^3}$**

**Solution:**

$$x - \frac{1}{2x} = 6$$

**To Find:**

$$x^3 - \frac{1}{8x^3} = ?$$

As we have

$$\left(x - \frac{1}{2x}\right)^3 = x^3 - \frac{1}{8x^3} - 3(x)\left(\frac{1}{2x}\right)\left(x - \frac{1}{2x}\right)$$

Put the values

$$(6)^3 = x^3 - \frac{1}{8x^3} - \frac{3}{2}(6)$$

$$216 = x^3 - \frac{1}{8x^3} - 3(3)$$

$$216 = x^3 - \frac{1}{8x^3} - 9$$

Add 9 on B.S

$$216 + 9 = x^3 - \frac{1}{8x^3} - 9 + 9$$

$$225 = x^3 - \frac{1}{8x^3}$$

$$x^3 - \frac{1}{8x^3} = 225$$

**Q7:** **If $a + b = 6$, show that $a^3 + b^3 + 18ab = 216$.**

**Solution:**

$$a + b = 6$$

**To Prove:**

$$a^3 + b^3 + 18ab = 216$$

As we have

$$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

Put the values

$$(6)^3 = a^3 + b^3 + 3ab(6)$$

$$216 = a^3 + b^3 + 18ab$$

$$a^3 + b^3 + 18ab = 216$$

**Q8:** **If $u - v = 3$ then prove that $u^3 - v^3 - 9uv = 27$.**

**Solution:**

$$u - v = 3$$

**To Prove:**

$$u^3 - v^3 - 9uv = 27$$

As we have

$$(u - v)^3 = u^3 - v^3 - 3uv(u - v)$$

### Ex # 4.5

Put the values

$$(2)^3 = a^3 - b^3 - 3(15)(2)$$

$$8 = a^3 - b^3 - 90$$

Add 90 on B.S

$$8 + 90 = a^3 - b^3 - 90 + 90$$

$$98 = a^3 - b^3$$

$$a^3 - b^3 = 98$$

**Q9:** **If $a + \frac{1}{a} = 2$, find the values of $a^2 + \frac{1}{a^2}$, $a^4 + \frac{1}{a^4}$, $a^3 + \frac{1}{a^3}$**

**Solution:**

**Given**

$$a + \frac{1}{a} = 2$$

**To prove**

$$a^2 + \frac{1}{a^2} = ?$$

$$a^4 + \frac{1}{a^4} = ?$$

$$a^3 + \frac{1}{a^3} = ?$$

As we have

$$\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2(a)\left(\frac{1}{a}\right)$$

Put the values

$$(2)^2 = a^2 + \frac{1}{a^2} + 2$$

$$4 = a^2 + \frac{1}{a^2} + 2$$

Subtract 2 from B.S

$$4 - 2 = a^2 + \frac{1}{a^2} + 2 - 2$$

$$2 = a^2 + \frac{1}{a^2}$$

$$a^2 + \frac{1}{a^2} = 2$$

**Now take square on B.S**

$$\left(a^2 + \frac{1}{a^2}\right)^2 = (2)^2$$

$$(a^2)^2 + \left(\frac{1}{a^2}\right)^2 + 2(a^2)\left(\frac{1}{a^2}\right) = 4$$

$$a^4 + \frac{1}{a^4} + 2 = 4$$

# Chapter # 4

**Ex # 4.5**

Subtract 2 from B.S

$a^4 + \frac{1}{a^4} + 2 - 2 = 4 - 2$

$a^4 + \frac{1}{a^4} = 2$

***Now*** $\boldsymbol{a^3 + \frac{1}{a^3}}$

Also we have

$\boldsymbol{\left(a + \frac{1}{a}\right)^3 = a^3 + \frac{1}{a^3} + 3(a)\left(\frac{1}{a}\right)\left(a + \frac{1}{a}\right)}$

Put the values

$(2)^3 = a^3 + \frac{1}{a^3} + 3(2)$

$8 = a^3 + \frac{1}{a^3} + 6$

Subtract 6 from B.S

$8 - 6 = a^3 + \frac{1}{a^3} + 6 - 6$

$2 = a^3 + \frac{1}{a^3}$

$a^3 + \frac{1}{a^3} = 2$

Hence

$$a^2 + \frac{1}{a^2} = a^4 + \frac{1}{a^4} = a^3 + \frac{1}{a^3} = 2$$

**<u>Ex # 4.6</u>**

1. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
2. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
3. $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - (x)\left(\frac{1}{x}\right)\right)$
4. $x^3 - \frac{1}{x^3} = \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + (x)\left(\frac{1}{x}\right)\right)$

OR

1. $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - 1\right)$
2. $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - 1\right)$
3. $(x + y)(x^2 - xy + y^2) = x^3 + y^3$
4. $(x - y)(x^2 + xy + y^2) = x^3 - y^3$
5. $(x + y)(x - y) = x^2 - y^2$

## Ex # 4.6

**Page # 118**

**Q1: Find the following product.**

**(i)** $\boldsymbol{(a - 1)(a^2 + a + 1)}$

**<u>Solution:</u>**

$(a - 1)(a^2 + a + 1)$

$= (a - 1)[(a)^2 + (a)(1) + (1)^2]$

As we know that

$\boldsymbol{(a - b)(a^2 + ab + b^2) = a^3 - b^3}$

Here $a = a$ and $b = 1$

So

$= (a)^3 - (1)^3$

$= a^3 - 1$

**(ii)** $\boldsymbol{(3 - b)(9 + 3b + b^2)}$

**<u>Solution:</u>**

$(3 - b)(9 + 3b + b^2)$

$= (3 - b)[(3)^2 + (3)(b) + (b)^2]$

As we know that

$\boldsymbol{(a - b)(a^2 + ab + b^2) = a^3 - b^3}$

Here $a = 3$ and $b = b$

So

$= (3)^3 - (b)^3$

$= 27 - b^3$

**(iii)** $\boldsymbol{(8 + b)(64 - 8b + b^2)}$

**<u>Solution:</u>**

$(8 + b)(64 - 8b + b^2)$

$= (8 + b)[(8)^2 - (8)(b) + (b)^2]$

As we know that

$\boldsymbol{(a + b)(a^2 - ab + b^2) = a^3 + b^3}$

Here $a = 8$ and $b = b$

So

$= (8)^3 + (b)^3$

$= 512 + b^3$

**(iv)** $(a + 2)(a^2 - 2a + 4)$

**<u>Solution:</u>**

$(a + 2)(a^2 - 2a + 4)$

$= (a + 2)[(a)^2 - (a)(2) + (2)^2]$

As we know that

$\boldsymbol{(a + b)(a^2 - ab + b^2) = a^3 + b^3}$

Here $a = a$ and $b = 2$

So

$= (a)^3 + (2)^3$

$= a^3 + 8$

# Chapter # 4



## Ex # 4.6

**Q2: Find the following product.**

**(i)** $\left(2p+\frac{1}{2p}\right)\left(4p^2+\frac{1}{4p^2}-1\right)$

**Solution:**

$$\left(2p+\frac{1}{2p}\right)\left(4p^2+\frac{1}{4p^2}-1\right)$$

$$\left(2p+\frac{1}{2p}\right)\left[(2p)^2+\frac{1}{(2p)^2}-(2p)\left(\frac{1}{2p}\right)\right]$$

As we know that

$$\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-(x)\left(\frac{1}{x}\right)\right)=x^3+\frac{1}{x^3}$$

So

$$=(2p)^3+\left(\frac{1}{2p}\right)^3$$

$$=8p^3+\frac{1}{8p^3}$$

**(ii)** $\left(\frac{3}{2}p-\frac{2}{3p}\right)\left(\frac{9}{4}p^2+\frac{4}{9p^2}+1\right)$

**Solution:**

$$\left(\frac{3}{2}p-\frac{2}{3p}\right)\left(\frac{9}{4}p^2+\frac{4}{9p^2}+1\right)$$

$$\left(\frac{3}{2}p-\frac{2}{3p}\right)\left[\left(\frac{3}{2}p\right)^2+\left(\frac{2}{3p}\right)^2+\left(\frac{3}{2}p\right)\left(\frac{2}{3p}\right)\right]$$

As we know that

$$\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+(x)\left(\frac{1}{x}\right)\right)=x^3-\frac{1}{x^3}$$

So

$$=\left(\frac{3}{2}p\right)^3-\left(\frac{2}{3p}\right)^3$$

$$=\frac{27}{8}p^3-\frac{8}{27p^3}$$

**(iii)** $\left(3p-\frac{1}{3p}\right)\left(9p^2+\frac{1}{9p^2}+1\right)$

**Solution:**

$$\left(3p-\frac{1}{3p}\right)\left(9p^2+\frac{1}{9p^2}+1\right)$$

$$\left(3p-\frac{1}{3p}\right)\left[(3p)^2+\frac{1}{(3p)^2}+(3p)\left(\frac{1}{3p}\right)\right]$$

As we know that

$$\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+(x)\left(\frac{1}{x}\right)\right)=x^3-\frac{1}{x^3}$$

## Ex # 4.6

So

$$=(3p)^3-\left(\frac{1}{3p}\right)^3$$

$$=27p^3+\frac{1}{27p^3}$$

**(iv)** $\left(5p+\frac{1}{5p}\right)\left(25p^2+\frac{1}{25p^2}-1\right)$

**Solution:**

$$\left(5p+\frac{1}{5p}\right)\left(25p^2+\frac{1}{25p^2}-1\right)$$

$$\left(5p+\frac{1}{5p}\right)\left[(5p)^2+\frac{1}{(5p)^2}-(5p)\left(\frac{1}{5p}\right)\right]$$

As we know that

$$\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-(x)\left(\frac{1}{x}\right)\right)=x^3+\frac{1}{x^3}$$

So

$$=(5p)^3+\left(\frac{1}{5p}\right)^3$$

$$=125p^3+\frac{1}{125p^3}$$

**Q3: Find the following continued product.**

**(i)** $(x^2-y^2)(x^2-xy+y^2)(x^2+xy+y^2)$

**Solution:**

$$(x^2-y^2)(x^2-xy+y^2)(x^2+xy+y^2)$$

**Using $a^2-b^2=(a+b)(a-b)$**

$$=(x+y)(x-y)(x^2-xy+y^2)(x^2+xy+y^2)$$

Arrange it

$$=(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)$$

**By Using Formulas**

$$=(x^3+y^3)(x^3-y^3)$$

**Again by Formula**

$$=(x^3)^2-(y^3)^2$$

$$=x^6-y^6$$

# Chapter # 4

**Ex # 4.6**

**(ii)** $(\boldsymbol{x+y})(\boldsymbol{x-y})(\boldsymbol{x^2+y^2})(\boldsymbol{x^4+y^4})$

**Solution:**

$(x+y)(x-y)(x^2+y^2)(x^4+y^4)$

**Using Formula** $(\boldsymbol{a+b})(\boldsymbol{a-b}) = \boldsymbol{a^2-b^2}$

$(x^2-y^2)(x^2+y^2)(x^4+y^4)$

**Again by Formula**

$[(x^2)^2-(y^2)^2](x^4+y^4)$

$(x^4-y^4)(x^4+y^4)$

**Now again by Formula**

$(x^4)^2-(y^4)^2$

$x^8-y^8$

**iii.** $(\boldsymbol{2x-y})(\boldsymbol{2x+y})(\boldsymbol{4x^2-2xy+y^2})(\boldsymbol{4x^2+2xy+y^2})$

**Solution:**

$(2x-y)(2x+y)(4x^2-2xy+y^2)(4x^2+2xy+y^2)$

Arrange it

$(2x-y)(4x^2+2xy+y^2)(2x+y)(4x^2-2xy+y^2)$

$(2x-y)[(2x)^2+(2x)(y)+(y)^2](2x+y)[(2x)^2-(2x)(y)+(y)^2]$

**As** $(\boldsymbol{x-y})(\boldsymbol{x^2+xy+y^2}) = \boldsymbol{x^3-y^3}$

**and** $(\boldsymbol{x+y})(\boldsymbol{x^2-xy+y^2}) = \boldsymbol{x^3+y^3}$

$[(2x)^3-(y)^3][(2x)^3+(y)^3]$

$(8x^3-y^3)(8x^3+y^3)$

**Using Formula** $(\boldsymbol{a+b})(\boldsymbol{a-b}) = \boldsymbol{a^2-b^2}$

$(8x^3)^2-(y^3)^2$

$64x^6-y^6$

**iv.** $(\boldsymbol{x-2})(\boldsymbol{x+2})(\boldsymbol{x^2-2x+4})(\boldsymbol{x^2+2x+4})$

**Solution:**

$(x-2)(x+2)(x^2-2x+4)(x^2+2x+4)$

Arrange it

$(x-2)(x^2+2x+4)(x+2)(x^2-2x+4)$

$(x-2)[(x)^2+(x)(2)+(2)^2](x+2)[(x)^2-(x)(2)+(2)^2]$

**As** $(\boldsymbol{x-y})(\boldsymbol{x^2+xy+y^2}) = \boldsymbol{x^3-y^3}$

**and** $(\boldsymbol{x+y})(\boldsymbol{x^2-xy+y^2}) = \boldsymbol{x^3+y^3}$

$[(x)^3-(2)^3][(x)^3+(2)^3]$

$(x^3-8)(x^3+8)$

**Using Formula** $(\boldsymbol{a+b})(\boldsymbol{a-b}) = \boldsymbol{a^2-b^2}$

$(x^3)^2-(8)^2$

$x^6-64$

**Ex # 4.6**

**Q4:** **Find the product with the help of formula.** $(\boldsymbol{\sqrt{x}-\sqrt{y}})(\boldsymbol{x+\sqrt{xy}+y})$

**Solution:**

$(\sqrt{x}-\sqrt{y})(x+\sqrt{xy}+y)$

$= (\sqrt{x}-\sqrt{y})\left[(\sqrt{x})^2+(\sqrt{x})(\sqrt{y})+(\sqrt{y})^2\right]$

**As** $(\boldsymbol{x-y})(\boldsymbol{x^2+xy+y^2}) = \boldsymbol{x^3-y^3}$

$= (\sqrt{x})^3-(\sqrt{y})^3$

$= \left(x^{\frac{1}{2}}\right)^3-\left(y^{\frac{1}{2}}\right)^3$

$= x^{\frac{3}{2}}-y^{\frac{3}{2}}$

**Q5:** **Simplify with the help of formula.** $(\boldsymbol{x^p+y^q})(\boldsymbol{x^{2p}-x^py^q+y^{2q}})$

**Solution:**

$(x^p+y^q)(x^{2p}-x^py^q+y^{2q})$

$= (x^p+y^q)[(x^p)^2-(x^p)(y^q)+(y^q)^2]$

**As** $(\boldsymbol{x+y})(\boldsymbol{x^2-xy+y^2}) = \boldsymbol{x^3+y^3}$

$= (x^p)^3+(y^p)^3$

$= x^{3p}+y^{3p}$

**Examples Page # 116 and 117**

# Chapter # 4

**Ex # 4.7**

**SURDS**

A number of the form of $\sqrt[n]{a}$ is called Surd, where $a$ is a positive rational number.

A number will be a surd, if

i. It is irrational
ii. It is a root
iii. A root of a rational number.

**Examples:**

$\sqrt{3}$ and $\sqrt{5+\sqrt{3}}$

In the above examples, both are irrational numbers. First number is a root of rational number 3, whereas the second number is a root of irrational number $5+\sqrt{3}$.

Thus $\sqrt{3}$ is a surd and $\sqrt{5+\sqrt{3}}$ is not a surd.

$\sqrt[3]{8}$ is not a surd because its value is 2 which is rational.

$\sqrt{-2}$ , $\sqrt{-3}$ are not surds because $-2$ *and* $-3$ are negative.

**Conjugate of Surds**

The conjugate of $a\sqrt{x}+b\sqrt{y}$ is $a\sqrt{x}-b\sqrt{y}$.

Similarly the conjugate of $5+\sqrt{3}$ is $5-\sqrt{3}$

## Ex # 4.7

**Page # 122**

**Q1: State which of the following are surd quantities**

**(i)** $\sqrt[3]{81}$

As 81 is a rational number and the result is irrational.
So it is surd.

**(ii)** $\sqrt{1+\sqrt{5}}$

As $1+\sqrt{5}$ is irrational.
So it is not surd.

**(iii)** $\sqrt{\sqrt{5}}$

As $\sqrt{5}$ is irrational.
So it is not surd.

**(iv)** $\sqrt[4]{32}$

As 32 is a rational number and the result is irrational.
So it is surd.

**Ex # 4.7**

**(v)** $\pi$

As $\pi$ is irrational.
So it is not surd.

**(vi)** $\sqrt{1+\pi^2}$

As $1+\pi^2$ is irrational.
So it is not surd.

**Q2: Express the following as the simplest possible surds.**

**(i)** $\sqrt{12}$

**Solution:**

$\sqrt{12}$

$\sqrt{2\times2\times3}$

$\sqrt{2^2\times3}$

$\sqrt{2^2}\sqrt{3}$

$2\sqrt{3}$

| | |
|---|---|
| 2 | 12 |
| 2 | 6 |
| 3 | 3 |
| | 1 |

**(ii)** $\sqrt{48}$

**Solution:**

$\sqrt{48}$

$\sqrt{2\times2\times2\times2\times3}$

$\sqrt{2^2\times2^2\times3}$

$\sqrt{2^2}\sqrt{2^2}\sqrt{3}$

$2\times2\sqrt{3}$

$4\sqrt{3}$

| | |
|---|---|
| 2 | 48 |
| 2 | 24 |
| 2 | 12 |
| 2 | 6 |
| 3 | 3 |
| | 1 |

**(iii)** $\sqrt{240}$

**Solution:**

$\sqrt{240}$

$\sqrt{2\times2\times2\times2\times3\times5}$

$\sqrt{2^2\times2^2\times3\times5}$

$\sqrt{2^2}\sqrt{2^2}\sqrt{3\times5}$

$2\times2\sqrt{15}$

$4\sqrt{15}$

| | |
|---|---|
| 2 | 240 |
| 2 | 120 |
| 2 | 60 |
| 2 | 30 |
| 3 | 15 |
| 5 | 5 |
| | 1 |

# Chapter # 4

**Ex # 4.7**

**Q3: Simplify the following surds.**

**(i)** $\left(\mathbf{2-\sqrt{3}}\right)\left(\mathbf{3+\sqrt{5}}\right)$

**Solution:**

$\left(2-\sqrt{3}\right)\left(3+\sqrt{5}\right)$

$2\left(3+\sqrt{5}\right)-\sqrt{3}\left(3+\sqrt{5}\right)$

$6+2\sqrt{5}-3\sqrt{3}-\sqrt{3\times 5}$

$6+2\sqrt{5}-3\sqrt{3}-\sqrt{15}$

**(ii)** $\left(\mathbf{\sqrt{3}-4}\right)\left(\mathbf{\sqrt{2}+1}\right)$

**Solution:**

$\left(\sqrt{3}-4\right)\left(\sqrt{2}+1\right)$

$\sqrt{3}\left(\sqrt{2}+1\right)-4\left(\sqrt{2}+1\right)$

$\sqrt{3\times 2}+1\sqrt{3}-4\sqrt{2}-4$

$\sqrt{6}+\sqrt{3}-4\sqrt{2}-4$

**(iii)** $\left(\mathbf{\sqrt{2}+\sqrt{3}}\right)\left(\mathbf{\sqrt{5}+\sqrt{2}}\right)$

**Solution:**

$\left(\sqrt{2}+\sqrt{3}\right)\left(\sqrt{5}+\sqrt{2}\right)$

$\sqrt{2}\left(\sqrt{5}+\sqrt{2}\right)+\sqrt{3}\left(\sqrt{5}+\sqrt{2}\right)$

$\sqrt{2\times 5}+\sqrt{2\times 2}+\sqrt{3\times 5}+\sqrt{3\times 2}$

$\sqrt{10}+2+\sqrt{15}+\sqrt{6}$

**(iv)** $\left(\mathbf{3-2\sqrt{3}}\right)\left(\mathbf{3+2\sqrt{3}}\right)$

**Solution:**

$\left(3-2\sqrt{3}\right)\left(3+2\sqrt{3}\right)$

Using Formula: $(a+b)(a+b)=a^2-b^2$

So

$(3)^2-\left(2\sqrt{3}\right)^2$

$9-(2)^2\left(\sqrt{3}\right)^2$

$9-4(3)$

$9-12$

$-3$

**Q4: Rationalize the denominator and simplify.**

**(i)** $\dfrac{\mathbf{1}}{\mathbf{\sqrt{7}}}$

**Solution:**

$\dfrac{1}{\sqrt{7}}$

Multiply and divide by $\sqrt{7}$

$\dfrac{1}{\sqrt{7}}\times\dfrac{\sqrt{7}}{\sqrt{7}}$

$\dfrac{1\sqrt{7}}{\left(\sqrt{7}\right)^2}$

$\dfrac{\sqrt{7}}{7}$

**(ii)** $\dfrac{\mathbf{3}}{\mathbf{\sqrt{45}}}$

**Solution:**

$\dfrac{3}{\sqrt{45}}$

$\dfrac{3}{\sqrt{3\times 3\times 5}}$

$\dfrac{3}{3\sqrt{5}}$

$\dfrac{1}{\sqrt{5}}$

Multiply and divide by $\sqrt{5}$

$\dfrac{1}{\sqrt{5}}\times\dfrac{\sqrt{5}}{\sqrt{5}}$

$\dfrac{1\sqrt{5}}{\left(\sqrt{5}\right)^2}$

$\dfrac{\sqrt{5}}{5}$

**(iii)** $\dfrac{\mathbf{1}}{\mathbf{\sqrt{2}-1}}$

**Solution:**

$\dfrac{1}{\sqrt{2}-1}$

Multiply and divide by $\sqrt{2}+1$

$\dfrac{1}{\sqrt{2}-1}\times\dfrac{\sqrt{2}+1}{\sqrt{2}+1}$

$\dfrac{1\left(\sqrt{2}+1\right)}{\left(\sqrt{2}\right)^2-(1)^2}$

$\dfrac{\sqrt{2}+1}{2-1}$

$\sqrt{2}+1$

# Chapter # 4

## Ex # 4.7

**(iv)** $\mathbf{\dfrac{5}{2+\sqrt{5}}}$

**Solution:**

$\dfrac{5}{2+\sqrt{5}}$

Multiply and divide by $2-\sqrt{5}$

$\dfrac{5}{2+\sqrt{5}}\times\dfrac{2-\sqrt{5}}{2-\sqrt{5}}$

$\dfrac{5(2-\sqrt{5})}{(2)^2-(\sqrt{5})^2}$

$\dfrac{5(2-\sqrt{5})}{4-5}$

$\dfrac{5(2-\sqrt{5})}{-1}$

$-5(2-\sqrt{5})$

**(v)** $\mathbf{\dfrac{1}{\sqrt{5}-2}+\dfrac{1}{\sqrt{5}+2}}$

**Solution:**

$\dfrac{1}{\sqrt{5}-2}+\dfrac{1}{\sqrt{5}+2}$

$\dfrac{1(\sqrt{5}+2)+1(\sqrt{5}-2)}{(\sqrt{5}-2)(\sqrt{5}+2)}$

$\dfrac{\sqrt{5}+2+\sqrt{5}-2}{(\sqrt{5})^2-(2)^2}$

$\dfrac{\sqrt{5}+\sqrt{5}}{5-4}$

$\dfrac{2\sqrt{5}}{1}$

$2\sqrt{5}$

**Q5:** **If $x=\sqrt{5}+2$, find the value of $x+\dfrac{1}{x}$ and $x^2+\dfrac{1}{x^2}$**

**Solution:**

$x=\sqrt{5}+2$

**To find:**

$$x+\frac{1}{x}=?\text{ and }x^2+\frac{1}{x^2}=?$$

$\dfrac{1}{x}=\dfrac{1}{\sqrt{5}+2}$

## Ex # 4.7

**Multiply and divide by $\sqrt{5}-2$**

$\dfrac{1}{x}=\dfrac{1}{\sqrt{5}+2}\times\dfrac{\sqrt{5}-2}{\sqrt{5}-2}$

$\dfrac{1}{x}=\dfrac{1(\sqrt{5}-2)}{(\sqrt{5}+2)(\sqrt{5}-2)}$

$\dfrac{1}{x}=\dfrac{\sqrt{5}-2}{(\sqrt{5})^2-(2)^2}$

$\dfrac{1}{x}=\dfrac{\sqrt{5}-2}{5-4}$

$\dfrac{1}{x}=\dfrac{\sqrt{5}-2}{1}$

$\dfrac{1}{x}=\sqrt{5}-2$

Now

$x+\dfrac{1}{x}=(\sqrt{5}+2)+(\sqrt{5}-2)$

$x+\dfrac{1}{x}=\sqrt{5}+2+\sqrt{5}-2$

$x+\dfrac{1}{x}=2\sqrt{5}$

Taking Square on B.S

$\left(x+\dfrac{1}{x}\right)^2=(2\sqrt{5})^2$

$x^2+\dfrac{1}{x^2}+2(x)\left(\dfrac{1}{x}\right)=(2)^2(\sqrt{5})^2$

$x^2+\dfrac{1}{x^2}+2=4(5)$

$x^2+\dfrac{1}{x^2}+2=20$

Subtract 2 from B.S

$x^2+\dfrac{1}{x^2}+2-2=20-2$

$x^2+\dfrac{1}{x^2}=18$

**Answers:**

$$x+\frac{1}{x}=2\sqrt{5}$$

$$x^2+\frac{1}{x^2}=18$$

**Ex # 4.7**

**Q6:** **If $x = \sqrt{2}+\sqrt{3}$, find the value of $x-\frac{1}{x}$ and $x^2+\frac{1}{x^2}$**

**Solution:**

$x = \sqrt{2}+\sqrt{3}$

**To find:**

$$x-\frac{1}{x}=?\text{ and } x^2+\frac{1}{x^2}=?$$

$\frac{1}{x}=\frac{1}{\sqrt{2}+\sqrt{3}}$

**Multiply and divide by $\sqrt{2}-\sqrt{3}$**

$\frac{1}{x}=\frac{1}{\sqrt{2}+\sqrt{3}}\times\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$

$\frac{1}{x}=\frac{1(\sqrt{2}-\sqrt{3})}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})}$

$\frac{1}{x}=\frac{\sqrt{2}-\sqrt{3}}{(\sqrt{2})^2-(\sqrt{3})^2}$

$\frac{1}{x}=\frac{\sqrt{2}-\sqrt{3}}{2-3}$

$\frac{1}{x}=\frac{\sqrt{2}-\sqrt{3}}{-1}$

$\frac{1}{x}=-(\sqrt{2}-\sqrt{3})$

$\frac{1}{x}=-\sqrt{2}+\sqrt{3}$

Now

$x-\frac{1}{x}=(\sqrt{2}+\sqrt{3})-(-\sqrt{2}+\sqrt{3})$

$x-\frac{1}{x}=\sqrt{2}+\sqrt{3}+\sqrt{2}-\sqrt{3}$

$x-\frac{1}{x}=2\sqrt{2}$

Taking Square on B.S

$\left(x-\frac{1}{x}\right)^2=(2\sqrt{2})^2$

$x^2+\frac{1}{x^2}-2(x)\left(\frac{1}{x}\right)=(2)^2(\sqrt{2})^2$

$x^2+\frac{1}{x^2}-2=4(2)$

$x^2+\frac{1}{x^2}-2=8$

**Ex # 4.7**

Add 2 on B.S

$x^2+\frac{1}{x^2}-2+2=8+2$

$x^2+\frac{1}{x^2}=10$

**Answers:**

$$x-\frac{1}{x}=2\sqrt{2}$$

$$x^2+\frac{1}{x^2}=10$$

**Q7:** **If $x = 5-2\sqrt{6}$, find the value of $x+\frac{1}{x}$ and $x^2+\frac{1}{x^2}$**

**Solution:**

$x=5-2\sqrt{6}$

**To find:**

$$x+\frac{1}{x}=?\text{ and } x^2+\frac{1}{x^2}=?$$

$\frac{1}{x}=\frac{1}{5-2\sqrt{6}}$

Multiply and divide by $5+2\sqrt{6}$

$\frac{1}{x}=\frac{1}{5-2\sqrt{6}}\times\frac{5+2\sqrt{6}}{5+2\sqrt{6}}$

$\frac{1}{x}=\frac{1(5+2\sqrt{6})}{(5-2\sqrt{6})(5+2\sqrt{6})}$

$\frac{1}{x}=\frac{5+2\sqrt{6}}{(5)^2-(2\sqrt{6})^2}$

$\frac{1}{x}=\frac{5+2\sqrt{6}}{25-(2)^2(\sqrt{6})^2}$

$\frac{1}{x}=\frac{5+2\sqrt{6}}{25-(4)(6)}$

$\frac{1}{x}=\frac{5+2\sqrt{6}}{25-24}$

$\frac{1}{x}=\frac{5+2\sqrt{6}}{1}$

$\frac{1}{x}=5+2\sqrt{6}$

**Ex # 4.7**

Now

$$x+\frac{1}{x}=\left(5-2\sqrt{6}\right)+\left(5+2\sqrt{6}\right)$$

$$x+\frac{1}{x}=5-2\sqrt{6}+5+2\sqrt{6}$$

$$x+\frac{1}{x}=10$$

Taking Square on B.S

$$\left(x+\frac{1}{x}\right)^2=(10)^2$$

$$x^2+\frac{1}{x^2}+2(x)\left(\frac{1}{x}\right)=100$$

$$x^2+\frac{1}{x^2}+2=100$$

Subtract 2 from B.S

$$x^2+\frac{1}{x^2}+2-2=100-2$$

$$x^2+\frac{1}{x^2}=98$$

**Answers:**

$$x+\frac{1}{x}=10$$

$$x^2+\frac{1}{x^2}=98$$

**Q8:** **If $x=\frac{1}{\sqrt{2}-1}$ find the value of $x-\frac{1}{x}$ and $x^2+\frac{1}{x^2}$**

**Solution:**

$$x=\frac{1}{\sqrt{2}-1}$$

To find

$$x-\frac{1}{x}=?\text{ and }x^2+\frac{1}{x^2}=?$$

Now

$$\frac{1}{x}=\sqrt{2}-1$$

$$\frac{1}{x}=\frac{1}{\sqrt{5}+2}$$

**Multiply and divide by $\sqrt{5}-2$**

$$\frac{1}{x}=\frac{1}{\sqrt{5}+2}\times\frac{\sqrt{5}-2}{\sqrt{5}-2}$$

**Ex # 4.7**

$$\frac{1}{x}=\frac{1\left(\sqrt{5}-2\right)}{\left(\sqrt{5}+2\right)\left(\sqrt{5}-2\right)}$$

$$\frac{1}{x}=\frac{\sqrt{5}-2}{\left(\sqrt{5}\right)^2-(2)^2}$$

$$\frac{1}{x}=\frac{\sqrt{5}-2}{5-4}$$

$$\frac{1}{x}=\frac{\sqrt{5}-2}{1}$$

$$\frac{1}{x}=\sqrt{5}-2$$

Now

$$x-\frac{1}{x}=\left(\sqrt{2}+1\right)-\left(\sqrt{2}-1\right)$$

$$x-\frac{1}{x}=\sqrt{2}+1-\sqrt{2}+1$$

$$x-\frac{1}{x}=2$$

Taking Square on B.S

$$\left(x-\frac{1}{x}\right)^2=(2)^2$$

$$x^2+\frac{1}{x^2}-2(x)\left(\frac{1}{x}\right)=(2)^2$$

$$x^2+\frac{1}{x^2}-2=4$$

Add 2 on B.S

$$x^2+\frac{1}{x^2}-2+2=4+2$$

$$x^2+\frac{1}{x^2}=6$$

**Answers:**

$$x-\frac{1}{x}=2$$

$$x^2+\frac{1}{x^2}=6$$

# Chapter # 4

**Ex # 4.7**

**Q9:** **If $x = \sqrt{10} + 3$, find the value of $x - \frac{1}{x}$ and $x^2 + \frac{1}{x^2}$**

**Solution:**

$x = \sqrt{10} + 3$

To find

$$x - \frac{1}{x} = ? \text{ and } x^2 + \frac{1}{x^2} = ?$$

Now

$\frac{1}{x} = \frac{1}{\sqrt{10} + 3}$

**Multiply and divide by $\sqrt{10} - 3$**

$\frac{1}{x} = \frac{1}{\sqrt{10} + 3} \times \frac{\sqrt{10} - 3}{\sqrt{10} - 3}$

$\frac{1}{x} = \frac{1(\sqrt{10} - 3)}{(\sqrt{10} + 3)(\sqrt{10} - 3)}$

$\frac{1}{x} = \frac{\sqrt{10} - 3}{(\sqrt{10})^2 - (3)^2}$

$\frac{1}{x} = \frac{\sqrt{10} - 3}{10 - 9}$

$\frac{1}{x} = \frac{\sqrt{10} - 3}{1}$

$\frac{1}{x} = \sqrt{10} - 3$

Now

$x - \frac{1}{x} = (\sqrt{10} + 3) - (\sqrt{10} - 3)$

$x - \frac{1}{x} = \sqrt{10} + 3 - \sqrt{10} + 3$

$x - \frac{1}{x} = \sqrt{10} - \sqrt{10} + 3 + 3$

$x - \frac{1}{x} = 6$

Taking Square on B.S

$\left(x - \frac{1}{x}\right)^2 = (6)^2$

$x^2 + \frac{1}{x^2} - 2(x)\left(\frac{1}{x}\right) = 36$

**Ex # 4.7**

$x^2 + \frac{1}{x^2} - 2 = 36$

Add 2 on B.S

$x^2 + \frac{1}{x^2} - 2 + 2 = 36 + 2$

$x^2 + \frac{1}{x^2} = 38$

**Answers:**

$$x - \frac{1}{x} = 6$$

$$x^2 + \frac{1}{x^2} = 38$$

**Q10:** **If $x = 2 - \sqrt{3}$, find the value of $x^4 + \frac{1}{x^4}$**

**Solution:**

$x = 2 - \sqrt{3}$

To find

$$x + \frac{1}{x} = ? \text{ and } x^2 + \frac{1}{x^2} = ?$$

Now

$\frac{1}{x} = \frac{1}{2 - \sqrt{3}}$

Multiply and divide by $2 + \sqrt{3}$

$\frac{1}{x} = \frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$

$\frac{1}{x} = \frac{1(2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})}$

$\frac{1}{x} = \frac{2 + \sqrt{3}}{(2)^2 - (\sqrt{3})^2}$

$\frac{1}{x} = \frac{2 + \sqrt{3}}{4 - 3}$

$\frac{1}{x} = \frac{2 + \sqrt{3}}{1}$

$\frac{1}{x} = 2 + \sqrt{3}$

Now

$x + \frac{1}{x} = (2 - \sqrt{3}) + (2 + \sqrt{3})$

$x + \frac{1}{x} = 2 - \sqrt{3} + 2 + \sqrt{3}$

# Chapter # 4

**Ex # 4.7**

$$x+\frac{1}{x}=2+2-\sqrt{3}+\sqrt{3}$$

$$x+\frac{1}{x}=4$$

Taking Square on B.S

$$\left(x+\frac{1}{x}\right)^2=(4)^2$$

$$x^2+\frac{1}{x^2}+2(x)\left(\frac{1}{x}\right)=16$$

$$x^2+\frac{1}{x^2}+2=16$$

Subtract 2 from B.S

$$x^2+\frac{1}{x^2}+2-2=16-2$$

$$x^2+\frac{1}{x^2}=14$$

Again take the square on B.S

$$\left(x^2+\frac{1}{x^2}\right)^2=(14)^2$$

$$x^4+\frac{1}{x^4}+2(x^2)\left(\frac{1}{x^2}\right)=196$$

$$x^4+\frac{1}{x^4}+2=196$$

Subtract 2 from B.S

$$x^4+\frac{1}{x^4}+2-2=196-2$$

$$x^4+\frac{1}{x^4}=194$$

**Answer:**

$$x^4+\frac{1}{x^4}=194$$

# Review Exercise # 4

**Page # 124**

**Q2:** **Simplify** $\mathbf{\frac{12x^4y^5}{25a^3b^4}\cdot\frac{15a^5b^4}{16x^7y^2}}$

**Solution:**

$$\frac{12x^4y^5}{25a^3b^4}\cdot\frac{15a^5b^4}{16x^7y^2}$$

$$\frac{3y^3}{5}\cdot\frac{3a^2}{4x^3}$$

$$\frac{9y^3a^2}{20x^3}$$

$$\frac{9a^2y^3}{20x^3}$$

**Q3:** **Evaluate** $\mathbf{\frac{2x-3}{x^2-x+1}}$ **for** $\mathbf{x=2}$

**Solution:**

$$\frac{2x-3}{x^2-x+1}$$

Put the value

$$\frac{2x-3}{x^2-x+1}=\frac{2(2)-3}{(2)^2-(2)+1}$$

$$\frac{2x-3}{x^2-x+1}=\frac{4-3}{4-2+1}$$

$$\frac{2x-3}{x^2-x+1}=\frac{1}{2+1}$$

$$\frac{2x-3}{x^2-x+1}=\frac{1}{3}$$

**Q4:** **Find the value of** $\mathbf{x^2+y^2}$ **and** $\mathbf{xy}$ **when** $\mathbf{x+y=7}$ , $\mathbf{x-y=3}$.

**Solution:**

$$x+y=7\ ,\ x-y=3$$

**To Find:**

$x^2+y^2=?$ and $xy=?$

$\underline{\mathbf{x^2+y^2}}$

As we have

$$\mathbf{(x+y)^2+(x-y)^2=2(x^2+y^2)}$$

Put the values

$$(7)^2+(3)^2=2(x^2+y^2)$$

$$49+9=2(x^2+y^2)$$

$$58=2(x^2+y^2)$$

## Review Ex # 4

Divide B.S by 2

$$\frac{58}{2} = \frac{2(x^2+y^2)}{2}$$

$29 = x^2 + y^2$

$29 = x^2 + y^2$

<u>***xy***</u>

As we have

$$(\boldsymbol{x}+\boldsymbol{y})^2 - (\boldsymbol{x}-\boldsymbol{y})^2 = 4\boldsymbol{xy}$$

Put the values

$(7)^2 - (3)^2 = 4xy$

$49 - 9 = 4xy$

$40 = 4xy$

Divide B.S by 4

$$\frac{40}{4} = \frac{4xy}{4}$$

$10 = xy$

$xy = 10$

**Q5: Find the value of $a+b+c$ when $a^2+b^2+c^2 = 43$ and $ab+bc+ca = 3$.**

<u>**Solution:**</u>

$a^2+b^2+c^2 = 43$ and $ab+bc+ca = 3$

**To Find:**

$$a+b+c = ?$$

As we know that

$$(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})^2 = \boldsymbol{a}^2+\boldsymbol{b}^2+\boldsymbol{c}^2 + 2(\boldsymbol{ab}+\boldsymbol{bc}+\boldsymbol{ca})$$

Put the values

$(a+b+c)^2 = 43 + 2(3)$

$(a+b+c)^2 = 43 + 6$

$(a+b+c)^2 = 49$

Taking square root on B.S

$\sqrt{(a+b+c)^2} = \sqrt{49}$

$a+b+c = 7$

**Q6: If $a+b+c = 6$ and $a^2+b^2+c^2 = 24$, then find the value of $ab+bc+ca$**

<u>**Solution:**</u>

$a+b+c = 6 \ and \ a^2+b^2+c^2 = 24$

**To Find:**

$$ab+bc+ca = ?$$

As we know that

$$(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})^2 = \boldsymbol{a}^2+\boldsymbol{b}^2+\boldsymbol{c}^2 + 2(\boldsymbol{ab}+\boldsymbol{bc}+\boldsymbol{ca})$$

Review Ex # 4

Put the values

$(6)^2 = 24 + 2(ab+bc+ca)$

$36 = 25 + 2(ab+bc+ca)$

Subtract 24 from B.S

$36 - 24 = 24 - 24 + 2(ab+bc+ca)$

$12 = 2(ab+bc+ca)$

Divide B.S by 2

$$\frac{12}{2} = \frac{2(ab+bc+ca)}{2}$$

$6 = ab+bc+ca$

$ab+bc+ca = 6$

**Q7: If $2x - 3y = 8$ and $xy = 2$, then find the values of $8x^3 - 27y^3$.**

<u>**Solution:**</u>

$2x - 3y = 8$ and $xy = 2$

**To Find:**

$$8x^3 - 27y^3 = ?$$

As we have

$$(\boldsymbol{2x}-\boldsymbol{3y})^3 = (\boldsymbol{2x})^3 - (\boldsymbol{3y})^3 - 3(\boldsymbol{2x})(\boldsymbol{3y})(\boldsymbol{2x}-\boldsymbol{3y})$$

Put the values

$(8)^3 = 8x^3 - 27y^3 - 18xy(8)$

$512 = 8x^3 - 27y^3 - 18(2)(8)$

$512 = 8x^3 - 27y^3 - 288$

Add 288 on B.S

$512 + 288 = 8x^3 - 27y^3 - 288 + 288$

$800 = 8x^3 - 27y^3$

$8x^3 - 27y^3 = 800$

# Chapter # 4

## Review Ex # 4

**Q8:** **Find the product** $\left(\frac{4}{5}x-\frac{5}{4x}\right)\left(\frac{16}{25}x^2-\frac{25}{16x^2}+1\right)$

**Solution:**

$$\left(\frac{4}{5}x-\frac{5}{4x}\right)\left(\frac{16}{25}x^2-\frac{25}{16x^2}+1\right)$$

$$\left(\frac{4}{5}x-\frac{5}{4x}\right)\left[\left(\frac{4}{5}x\right)^2+\left(\frac{5}{4x}\right)^2+\left(\frac{4}{5}x\right)\left(\frac{5}{4x}\right)\right]$$

As we know that

$$\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+(x)\left(\frac{1}{x}\right)\right)=x^3-\frac{1}{x^3}$$

$$=\left(\frac{4}{5}x\right)^3-\left(\frac{5}{4x}\right)^3$$

$$=\frac{64}{125}x^3-\frac{125}{64x^3}$$

**Q9:** **Find the value of** $x^3+\frac{1}{x^3}$ **, when** $x+\frac{1}{x}=8$

**Solution:**

$$x+\frac{1}{x}=8$$

**To Find:**

$$x^3+\frac{1}{x^3}=?$$

As we have

$$\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3(x)\left(\frac{1}{x}\right)\left(x+\frac{1}{x}\right)$$

Put the values

$$(8)^3=x^3+\frac{1}{x^3}+3(8)$$

$$512=x^3+\frac{1}{x^3}+24$$

Subtract 24 from B.S

$$512-24=x^3+\frac{1}{x^3}+24-24$$

$$488=x^3+\frac{1}{x^3}$$

$$x^3+\frac{1}{x^3}=488$$

## Review Ex # 4

**Think Trick**

**Q10:** **Simplify** $\frac{2x^2}{x^4-16}-\frac{x}{x^2-4}+\frac{1}{x+2}$

**Solution:**

$$\frac{2x^2}{x^4-16}-\frac{x}{x^2-4}+\frac{1}{x+2}$$

$$\frac{2x^2}{x^4-16}+\frac{1}{x+2}-\frac{x}{x^2-4}$$

$$\frac{2x^2}{(x^2)^2-(4)^2}+\frac{1}{x+2}-\frac{x}{(x+2)(x-2)}$$

$$\frac{2x^2}{(x^2+4)(x^2-4)}+\frac{1(x-2)-x}{(x+2)(x-2)}$$

$$\frac{2x^2}{(x^2+4)(x^2-4)}+\frac{x-2-x}{(x+2)(x-2)}$$

$$\frac{2x^2}{(x^2+4)(x^2-4)}+\frac{x-x-2}{x^2-4}$$

$$\frac{2x^2}{(x^2+4)(x^2-4)}+\frac{-2}{x^2-4}$$

$$\frac{2x^2}{(x^2+4)(x^2-4)}-\frac{2}{x^2-4}$$

$$\frac{2x^2-2(x^2+4)}{(x^2+4)(x^2-4)}$$

$$\frac{2x^2-2x^2-8}{(x^2)^2-(4)^2}$$

$$\frac{-8}{x^4-16}$$

Tehkals.com
Learn & Teach

# MATHEMATICS

## Class 9th (KPK)

## Chapter # 6 Algebraic Manipulations

NAME: ______________________________

F.NAME: ______________________________

CLASS:_____________ SECTION: _________

ROLL #: ______ SUBJECT: ______________

ADDRESS: ________________________________________

__________________________________________________

SCHOOL: ___________________________________________

# Chapter # 6

# UNIT # 6

## ALGEBRAIC MANIPULATIONS

### Ex # 6.1

**Highest Common Factor (H.C.F)**

The highest number of factors common to all given expressions or polynomials is called Highest Common Factor (H.C.F)

In other words, H.C.F of two or more polynomials is a polynomial of the highest degree, which divides exactly the given polynomials.

There are two methods for finding H.C.F.

(i) H.C.F by Factorization

(ii) H.C.F by Division

**H.C.F by Factorization**

In this method, first factorize all the given expressions

Then we take all possible common factors which is the H.C.F of the given expression.

**Example # 1**

Find H.C.F of $x^2-y^2,\ x^2-xy$

**Solution:**

$x^2-y^2,\ x^2-xy$

$x^2-y^2=(x+y)(x-y)$

And

$x^2-xy=x(x-y)$

Here $x-y$ is a common factor. Thus

$\mathrm{H.C.F}=\mathrm{x}-\mathrm{y}$

**Example # 2**

Find H.C.F of $ax^2+5ax+6a$, $ax^3+9ax^2+14ax\ \ and\ \ 15a(x^2-4)$

**Solution:**

$ax^2+5ax+6a,\ \ ax^3+9ax^2+14ax\ and$ $15a(x^2-4)$

$ax^2+5ax+6a=a(x^2+5x+6)$

$ax^2+5ax+6a=a(x^2+2x+3x+6)$

$ax^2+5ax+6a=a[x(x+2)+3(x+2)]$

$ax^2+5ax+6a=a(x+2)(x+3)$

**And**

$ax^3+9ax^2+14ax=ax(x^2+9x+14)$

$ax^3+9ax^2+14ax=ax(x^2+2x+7x+14)$

$ax^3+9ax^2+14ax=ax[x(x+2)+7(x+2)]$

$ax^3+9ax^2+14ax=ax(x+2)(x+7)$

| $(x^2)(6)=6x^2$ | |
|---|---|
| **Add** | **Multiply** |
| $+2x$ | $+2x$ |
| $+3x$ | $+3x$ |
| $\boldsymbol{+5x}$ | $\boldsymbol{6x^2}$ |

| $(x^2)(14)=14x^2$ | |
|---|---|
| **Add** | **Multiply** |
| $+2x$ | $+2x$ |
| $+7x$ | $+7x$ |
| $\boldsymbol{+9x}$ | $\boldsymbol{14x^2}$ |

### Ex # 6.1

**Now also**

$15a(x^2-4)=3\times 5.a[(x)^2-(2)^2]$

$15a(x^2-4)=3\times 5.a(x+2)(x-2)$

Here $a(x+2)$ is common in given three expressions.

$\mathrm{H.C.F}=\mathrm{a}(x+2)$

**Note:**

The H.C.F $\mathrm{a}(x+2)$ exactly divides all the given three expression

**H.C.F by Division Method**

**Dividend**

$$\begin{array}{r|l|l} x^2-x-6 & x^2-2x-3 & 1 \\ & \underline{\pm x^2\mp x\mp 6} & \\ & -x+3 & \end{array}$$

**Remainder**

**Divisor** **Quotient**

**Steps**

1 Write the expressions in descending order

2 Take the common from the expressions if any.

3 Divide higher degree polynomial by the polynomial of lower degree

4 Divide to that time till the degree of remainder is less than the degree of divisor.

5 Now bring down the divisor and divide by remainder BUT before this take the common from the remainder if any.

6 Repeat the above steps till the remainder is zero.

7 Last divisor is the H.C.F of the given polynomials.

**Note:**

**1** In H.C.F by division, if required, multiply the expression by a suitable integer to avoid fraction.

**2** To find the H.C.F of three polynomials, first find H.C.F of any two of them, then find H.C.F of this H.C.F and the third polynomial.

# Chapter # 6

## Ex # 6.1

**H.C.F by Division method in Urdu**

1. تمام variables کو descending order میں لکھیں گے۔
2. اگر کوئی common ہو تو پہلے common لینگے۔
3. بڑے expression کو چھوٹے expression پر divide کریں گے۔
4. اس کو اس وقت تک divide کرتے رہیں گے جب تک remainder میں power ہمارے ساتھ divisor کے power سے کم نہ آئے
5. پھر divisor کو نیچے لائیں گے اور remainder پر divide کریں گے لیکن اس سے پہلے remainder میں common لیں گے اگر ہو۔
6. ان steps کو اس وقت تک کرو گے جب تک remainder میں zero نہ آئے۔
6. آخری divisor ہمارے ساتھ H.C.F ہوگا۔

**Example # 3**

Find H.C.F of $2x^3+7x^2+4x-4$ and $2x^3+9x^2+11x+2$

**Solution:**

$2x^3+7x^2+4x-4$ and $2x^3+9x^2+11x+2$

$$
\begin{array}{r|l|l}
2x^3+7x^2+4x-4 & 2x^3+9x^2+11x+2 & 1 \\
 & \pm 2x^3 \pm 7x^2 \pm 4x \mp 4 & \\
\hline
2x^2+7x+6 & 2x^3+7x^2+4x-4 & x \\
 & \pm 2x^3 \pm 7x^2 \pm 6x & \\
\hline
-2 & -2x-4 & \textbf{Dividing by } -2 \\
\hline
x+2 & 2x^2+7x+6 & 2x+3 \\
 & \pm 2x^2 \pm 4x & \\
\hline
 & 3x+6 & \\
 & \pm 3x \pm 6 & \\
\hline
 & \times & 
\end{array}
$$

Hence H.C.F= $x+2$

**Note:**

**H.C.F by Factorization**

H.C.F of 24 and 32

Factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24

Factors of 24 = 1, 2, 4, 8, 16, 32

Common factors $= 1, 2, 4, 8$

$\text{H.C.F} = 8$

# Chapter # 6

## Ex # 6.1

**Example # 4**

**Find H.C.F of $x^3-6x^2+11x-6$ , $3x^3-5x^2+6x-4$ and $2x^3+9x^2+11x+2$**

**Solution:**

$x^3-6x^2+11x-6$ , $3x^3-5x^2+6x-4$ and $2x^3+9x^2+11x+2$

$$3x^3-5x^2+6x-4 \;\big)\; 3x^3+5x^2-6x-2 \;\big(\; 1$$
$$\pm 3x^3 \mp 5x^2 \pm 6x \mp 4$$
$$2\,\big)\; 10x^2-12x+2 \qquad \textbf{Dividing by 2}$$
$$5x^2-6x+1 \;\big)\; 3x^3-5x^2+6x-4 \;\big(\; 3x-7$$
$$\times 5 \qquad \textbf{Multiplying by 5}$$
$$15x^3-25x^2+30x-20$$
$$\pm 15x^3 \mp 18x^2 \pm 3x$$
$$-7x^2+27x-20$$
$$\times 5 \qquad \textbf{Multiplying by 5}$$
$$-35x^2+135x-100$$
$$\mp 35x^2 \pm 42x \mp 7$$
$$93\,\big)\; 93x-93 \qquad \textbf{Dividing by 93}$$
$$x-1 \;\big)\; 5x^2-6x+1 \;\big(\; 5x-1$$
$$\pm 5x^2 \mp 5x$$
$$-x+1$$
$$\mp x \pm 1$$
$$\times$$

Hence H.C.F$=x-1$

Now find the H.C.F of $x-1$ and $x^3-6x^2+11x-6$

$$x-1 \;\big)\; x^3-6x^2+11x-6 \;\big(\; x^2-5x+6$$
$$\pm x^3 \mp x^2$$
$$-5x^2+11x-6$$
$$\mp 5x^2 \pm 5x$$
$$6x-6$$
$$\pm 6x \mp 6$$
$$\times$$

Hence the required H.C.F of $x^3-6x^2+11x-6$ , $3x^3-5x^2+6x-4$ and $2x^3+9x^2+11x+2$ is $\boldsymbol{x-1}$

**Least Common Multiple (L.C.M)**

The polynomial of least degree which is divisible by the given polynomials.

There are two methods of finding L.C.M

(a) L.C.M by factorization

(b) L.C.M by formula

# Chapter # 6

## Ex # 6.1

(a) **L.C.M by factorization**

In this method, first factorize all the given expressions

Then find the L.C.M by given formula.

$\boldsymbol{L.C.M = common\ factor \times non - common\ factor}$

**Example # 5**

**Find L.C.M of $\boldsymbol{x^2 + 4x + 4}$ and $\boldsymbol{x^2 + 5x + 6}$**

**Solution:**

$x^2 + 4x + 4$ and $x^2 + 5x + 6$

$x^2 + 4x + 4 = (x)^2 + 2(x)(2) + (2)^2$

$x^2 + 4x + 4 = (x + 2)^2$

$x^2 + 4x + 4 = (x + 2)(x + 2)$

**Now**

$x^2 + 5x + 6 = x^2 + 2x + 3x + 6$

$x^2 + 5x + 6 = x(x + 2) + 3(x + 2)$

$x^2 + 5x + 6 = (x + 2)(x + 3)$

$Common\ Factor = x + 2$

$Non - common\ factor = (x + 2)(x + 3)$

$\boldsymbol{L.C.M = common\ factor \times non - common\ factor}$

$L.C.M = (x + 2)(x + 2)(x + 3)$

$L.C.M = (x + 2)^2(x + 3)$

**Example # 6**

**Find L.C.M of $\boldsymbol{x^2 - 4x + 3}$ , $\boldsymbol{x^2 - 3x + 2}$ and $\boldsymbol{x^2 - 5x + 6}$**

**Solution:**

$x^2 - 4x + 3$ , $x^2 - 3x + 2$ and $x^2 - 5x + 6$

$x^2 - 4x + 3 = x^2 - x - 3x + 3$

$x^2 - 4x + 3 = x(x - 1) - 3(x - 1)$

$x^2 - 4x + 3 = (x - 1)(x - 3)$….. (i)

**Now**

$x^2 - 3x + 2 = x^2 - x - 2x + 3$

$x^2 - 3x + 2 = x(x - 1) - 2(x - 1)$

$x^2 - 3x + 2 = (x - 1)(x - 2)$….. (ii)

**Now**

$x^2 - 5x + 6 = x^2 - 2x - 3x + 6$

$x^2 - 5x + 6 = x(x - 2) - 3(x + 2)$

$x^2 - 5x + 6 = (x - 2)(x - 3)$….. (iii)

$x - 1$ in expression (i)& (ii)

$x - 2$ in expression (ii)& (iii)

$x - 3$ in expression (i)& (iii)

Therefore:

$\boldsymbol{L.C.M = common\ factor \times non - common\ factor}$

$L.C.M = (x - 1)(x - 2)(x - 3) \times 1$

$L.C.M = (x - 1)(x - 2)(x - 3)$

## Ex # 6.1

**L.C.M Theorem:**

If A and B are given polynomials and their H.C.F and L.C.M are represented by $H$ and $L$ respectively, then

$$A \times B = H \times L$$

**Proof:**

Since $H$ is common factor of polynomial of $A$ and $B$, then it divides exactly $A$ and $B$. So

$\frac{A}{H} = a$

$A = Ha$… equ(i)

and

$\frac{B}{H} = b$

$B = Hb$… equ(ii)

As $a$ and $b$ have no common factor.

As we know that:

$\boldsymbol{L.C.M = common\ factor \times non - common\ factor}$

$L = H \times a \times b$

Multiply B.S by $H$

$L \times H = H \times a \times b \times H$

$L \times H = (Ha) \times (Hb)$

Put equ(i) and equ(ii), we get

$L \times H = A \times B$

Or

$H \times L = Product\ of\ two\ polynomials$

**Formula for L.C.M**

As $L \times H = A \times B$

$L = \frac{A \times B}{H}$

$L.C.M = \frac{Product\ of\ two\ polynomials}{H.C.F}$

# Chapter # 6

## Ex # 6.1

**Example # 7**

**Find L.C.M of $x^3-6x^2+11x-6$ and $x^3-4x+3$**

$x^3-6x^2+11x-6$ and $x^3-4x+3$

**Solution:**

$Let\ A = x^3-6x^2+11x-6$

$and\ B = x^3-4x+3$

As we have:

$$L.C.M = \frac{A\times B}{H.C.F} --equ\ (i)$$

First we find H.C.F

$$x^3-4x+3 \,\big)\, x^3-6x^2+11x-6 \,\big(\, 1$$
$$\pm x^3 \quad \mp 4x \pm 3$$
$$-3 \,\big)\, -6x^2+15x-9$$
$$2x^2-5x+3 \,\big)\, x^3-4x+3 \,\big(\, x+5$$
$$\times 2$$
$$2x^3-8x+6$$
$$\pm 2x^3 \pm 3x \quad \mp 5x^2$$
$$5x^2-11x+6$$
$$\times 2$$
$$10x^2-22x+12$$
$$\pm 10x^2 \mp 25x \pm 15$$
$$3 \,\big)\, 3x-3$$
$$x-1 \,\big)\, 2x^2-5x+3 \,\big(\, 2x-3$$
$$\pm 2x^2 \mp 2x$$
$$-3x+3$$
$$\mp 3x \pm 3$$
$$\times$$

$H.C.F = x-1$

Now put the values in equ (i)

$$L.C.M = \frac{(x^3-6x^2+11x-6)(x^3-4x+3)}{x-1}$$

Now by Simple Division

$$x-1 \,\big)\, \overline{x^3-6x^2+11x-6} \quad x^2-5x+6$$
$$\pm x^3 \mp x^2$$
$$-5x^2+11x-6$$
$$\mp 5x^2 \pm 5x$$
$$6x-6$$
$$\pm 6x \mp 6$$
$$\times$$

$So\ L.C.M = (x^2-5x+6)(x^3-4x+3)$

# Chapter # 6

## Ex # 6.1

**Example # 8**

**Find H.C.F and L.C.M of $3x^3 - 2x^2 - 3x + 2$ and $6x^3 - 7x^2 - x + 2$**

$3x^3 - 2x^2 - 3x + 2$ and $6x^3 - 7x^2 - x + 2$

**Solution:**

$Let\ A = 3x^3 - 2x^2 - 3x + 2$

$and\ B = 6x^3 - 7x^2 - x + 2$

As we have:

$$L.C.M = \frac{A \times B}{H.C.F} - -equ\ (i)$$

First we find H.C.F

$$3x^3 - 2x^2 - 3x + 2 \;\Big)\; 6x^3 - 7x^2 - x + 2 \;\Big(\; 2$$
$$\pm 6x^3 \mp 4x^2 \mp 6x \pm 4$$
$$-1 \;\Big)\; -3x^2 + 5x - 2$$
$$3x^2 - 5x + 2 \;\Big)\; 3x^3 - 2x^2 - 3x + 2 \;\Big(\; x + 1$$
$$\pm 3x^3 \mp 5x^2 \pm 2x$$
$$3x^2 - 5x + 2$$
$$\pm 3x^2 \mp 5x \pm 2$$
$$\times$$

$$H.C.F = 3x^2 - 5x + 2$$

Now put the values in equ (i)

$$L.C.M = \frac{(3x^3 - 2x^2 - 3x + 2)(6x^3 - 7x^2 - x + 2)}{3x^2 - 5x + 2}$$

Now by Simple Division

$$x + 1$$
$$3x^2 - 5x + 2 \;\Big)\; 3x^3 - 2x^2 - 3x + 2$$
$$\pm 3x^3 \mp 5x^2 \pm 2x$$
$$3x^2 - 5x + 2$$
$$\pm 3x^2 \mp 5x \pm 2$$
$$\times$$

$So\ L.C.M = (x + 1)(6x^3 - 7x^2 - x + 2)$

**Example # 9**

**If H.C.F and L.C.M of two polynomials are $x - 3$ and $x^3 - 9x^2 + 26x - 24$ respectively. Find the second polynomial when one polynomial is $x^2 - 5x + 6$.**

**Solution:**

$H.C.F = x - 3$

$L.C.M = x^3 - 9x^2\_26x - 24$

$Let\ First\ polynomial = A = x^2 - 5x + 6$

$Second\ polynomial = B = ?$

As we have:

$$L.C.M = \frac{A \times B}{H.C.F}$$

$A \times B = L.C.M \times H.C.F$

# Chapter # 6

**Ex # 6.1**

$$B = \frac{L.C.M \times H.C.F}{A}$$

Put the values

$$B = \frac{(x^3 - 9x^2\_26x - 24)(x - 3)}{x^2 - 5x + 6}$$

Now by simple Division

$$\begin{array}{r|l} & \quad x - 4 \\ \hline x^2 - 5x + 6 & x^3 - 9x^2 + 26\,x - 24 \\ & \pm x^3 \mp 5x^2 \pm 6x \\ \hline & \quad -4x^2 + 20x - 24 \\ & \quad \mp 4x^2 \pm 20x \mp 24 \\ \hline & \qquad \times \end{array}$$

$So\ B = (x - 4)(x - 3)$
$B = x^2 - 3x - 4x + 12$
$B = x^2 - 7x + 12$
Hence the second polynomial is $x^2 - 7x + 12$

**Example # 10**

**If H.C.F and L.C.M of two polynomials are $x - 1$ and $x^3 + 4x^2 + x - 6$ respectively. Find the polynomials of degree 2.**

**Solution:**

$H.C.F = x - 1$
$L.C.M = x^3 + 4x^2 + x - 6$
$First\ polynomial = A = ?$
$Second\ polynomial = B = ?$
$As\ H.C.F = x - 1$
$then\ it\ is\ also\ the\ factor\ of\ L.C.M$
Now

$$\begin{array}{r|l} & \quad x^2 + 5x + 6 \\ \hline x - 1 & x^3 + 4x^2 + x - 6 \\ & \pm x^3 \mp x^2 \\ \hline & \quad 5x^2 + x - 6 \\ & \quad \pm 5x^2 \mp 5x \\ \hline & \quad 6x - 6 \\ & \quad \pm 6x \mp 6 \\ \hline & \qquad \times \end{array}$$

$L.C.M = x^3 + 4x^2 + x - 6$
$L.C.M = (x - 1)(x^2 + 5x + 6)$
$L.C.M = (x - 1)(x^2 + 3x + 2x + 6)$
$L.C.M = (x - 1)[x(x + 3) + 2(x + 3)]$
$L.C.M = (x - 1)(x + 3)(x + 2)$
$As\ x - 1\ is\ common\ factor. So$
$A = (x - 1)(x + 3)$

**Ex # 6.1**

$A = x^2 + 2x - 3$
And
$B = (x - 1)(x + 2)$
$B = x^2 + 2x - 1x - 2$
$B = x^2 + x - 2$

**Example # 11**

**The sum of two numbers is 120 and their H.C.F is 12. Find the numbers.**

**Solution:**

Let $x$ and $y$ be the two numbers.
As H.C.F is 12, means 12 is common factor.
So, it becomes
$12x + 12y = 120$
$12(x + y) = 120$
Divide B.S by 12, we get
$x + y = 12$
As the sum of two numbers is 10, so the possible pairs of numbers are (1,9), (2,8), (3,7), (4,6), (5,5)
As (1,9), (3,7) are non commo factors
Then the required numbers are:
$1 \times 12 = 12$ and $9 \times 12 = 108$
OR
$3 \times 12 = 36$ and $7 \times 12 = 84$

# Chapter # 6

## Exercise# 6.1

**Page # 159-160**

**Q1: 159** **Find H.C.F of the following expression by factorization method.**

**(i)** $(x+y)^2$ **and** $x^2-36$

**Solution:**

$(x+y)^2$ and $x^2-36$

$(x+y)^2=(x+y)(x+y)$

And

$x^2-36=(x)^2-(6)^2$

$=(x+6)(x-6)$

$H.C.F=x-6$

**(iii)** $x-3, x^2-9, (x-3)^2$

**Solution:**

$x-3, x^2-9, (x-3)^2$

$x-3=x-3$

And

$x^2-9=(x)^2-(3)^2$

$=(x+3)(x-3)$

And

$(x-3)^2=(x-3)(x-3)$

$H.C.F=\ x-3$



**(ii)** $x^4-y^4$ **and** $x^4+2x^2y^2+y^4$

**Solution:**

$x^4-y^4$ and $x^4+2x^2y^2+y^4$

$x^4-y^4=(x^2)^2-(y^2)^2$

$=(x^2+y^2)(x^2-y^2)$

$=(x^2+y^2)(x+y)(x-y)$

And

$x^4+2x^2y^2+y^4=(x^2)^2+2(x^2)(y^2)+(y^2)^2$

$=(x^2+y^2)^2$

$=(x^2+y^2)(x^2+y^2)$

$H.C.F=x^2+y^2$

**(v)** $2x^4-2y^4, 6x^2+12xy+6y^2, 9x^3+9y^3$

**Solution:**

$2x^4-2y^4, 6x^2+12xy+6y^2, 9x^3+9y^3$

$2x^4-2y^4=2[(x^2)^2-(y^2)^2]$

$=2(x^2+y^2)(x^2-y^2)$

$=2(x^2+y^2)(x+y)(x-y)$

And

$6x^2+12xy+6y^2=6(x^2+2xy+y^2)$

$=2\times 3(x+y)^2$

$=2\times 3(x+y)(x+y)$

And

$9x^3+9y^3=9(x^3+y^3)$

$=9(x+y)(x^2-xy+y^2)$

$H.C.F=x+y$

**(iv)** $2^3 3^2(x-y)^3(x+2y)^2, 2^3 3^2(x-y)^2(x+2y)^3, 3^2(x-y)^2(x+2y)$

**Solution:**

$2^3 3^2(x-y)^3(x+2y)^2, 2^3 3^2(x-y)^2(x+2y)^3, 3^2(x-y)^2(x+2y)$

$2^3 3^2(x-y)^3(x+2y)^2=2.2.2.3.3(x-y)(x-y)(x-y)(x+2y)(x+2y)$

$2^3 3^2(x-y)^2(x+2y)^3=2.2.2.3.3(x-y)(x-y)(x+2y)(x+2y)(x+2y)$

$3^2(x-y)^2(x+2y)=3.3(x-y)(x-y)(x+2y)$

$H.C.F=3.3(x-y)(x-y)(x+2y)$

$H.C.F=3^2(x-y)^2(x+2y)$

# Chapter # 6

**Ex # 6.1**

**Q2: Find H.C.F by division method.**

**160**

**(i)** $\boldsymbol{x^2 - x - 6}$ **and** $\boldsymbol{x^2 - 2x - 3}$

**Solution:**

$x^2 - x - 6$ and $x^2 - 2x - 3$

$$x^2 - x - 6 \;\big)\; x^2 - 2x - 3 \;\big(\; 1$$
$$\pm x^2 \mp x \mp 6$$
$$-1 \;\big)\; -x + 3$$
$$x - 3 \;\big)\; x^2 - x - 6 \;\big(\; x + 2$$
$$\pm x^2 \mp 3x$$
$$2x - 6$$
$$\pm 2x \mp 6$$
$$\times$$

$H.C.F = x - 3$

**(ii)** $\boldsymbol{y^3 - 3y + 2}$ **and** $\boldsymbol{y^3 - 5y^2 + 7y - 3}$

**Solution:**

$y^3 - 3y + 2$ and $y^3 - 5y^2 + 7y - 3$

$$y^3 - 3y + 2 \;\big)\; y^3 - 5y^2 + 7y - 3 \;\big(\; 1$$
$$\pm y^3 \mp 3y \pm 2$$
$$-5 \;\big)\; -5y^2 + 10y - 5$$
$$y^2 - 2y + 1 \;\big)\; y^3 - 3y + 2 \;\big(\; y + 2$$
$$\pm y^3 \pm 1y \mp 2y^2$$
$$2y^2 - 4y + 2$$
$$\pm 2y^2 \mp 4y \pm 2$$
$$\times$$

$H.C.F = y^2 - 2y + 1$

# Chapter # 6

## Ex # 6.1

**(iii)** $\mathbf{2x^5 - 4x^4 - 6x}$ **and** $\mathbf{x^5 + x^4 - 3x^3 - 3x^2}$

**Solution:**

$2x^5 - 4x^4 - 6x$ and $x^5 + x^4 - 3x^3 - 3x^2$

$2x^5 - 4x^4 - 6x = 2x(x^4 - 2x^3 - 3)$

$x^5 + x^4 - 3x^3 - 3x^2 = x^2(x^3 + x^2 - 3x - 3)$

$= x.x(x^3 + x^2 - 3x - 3)$

$$x^3 + x^2 - 3x - 3 \;\big)\; x^4 - 2x^3 - 3 \;\big(\; x$$
$$\pm x^4 \pm x^3 \quad \mp 3x^2 \mp 3x$$
$$-3 \;\big)\; -3x^3 + 3x^2 + 3x - 3$$
$$x^3 - x^2 - x + 1 \;\big)\; x^3 + x^2 - 3x - 3 \;\big(\; 1$$
$$\pm x^3 \mp x^2 \mp x \pm 1$$
$$2 \;\big)\; 2x^2 - 2x - 4$$
$$x^2 - x - 2 \;\big)\; x^3 - x^2 - x + 1 \;\big(\; x$$
$$\pm x^3 \mp x^2 \mp 2x$$
$$x + 1 \;\big)\; x^2 - x - 2 \;\big(\; x - 2$$
$$\pm x^2 \pm x$$
$$-2x - 2$$
$$\mp 2x \mp 2$$
$$\times$$

$H.C.F = x(x + 1)$



**(iv)** $\mathbf{2x^3 + 10x^2 + 5x + 25}$ **and** $\mathbf{x^3 + 5x^2 - x - 5}$

**Solution:**

$2x^3 + 10x^2 + 5x + 25$ and $x^3 + 5x^2 - x - 5$

$$x^3 + 5x^2 - x - 5 \;\big)\; 2x^3 + 10x^2 + 5x + 25 \;\big(\; 2$$
$$\pm 2x^3 \pm 10x^2 \mp 2x \mp 10$$
$$7 \;\big)\; 7x + 35$$
$$x + 5 \;\big)\; x^3 + 5x^2 - x - 5 \;\big(\; x^2 - 1$$
$$\pm x^3 \mp 5x^2$$
$$-x - 5$$
$$\mp x \mp 5$$
$$\times$$

$H.C.F = x + 5$

# Chapter # 6

## Ex # 6.1

**Q3: Find L.C.M by factorization.**

**(i)** $x+y, \quad x^2-y^2$

**Solution:**

$x+y, \quad x^2-y^2$

$x+y = x+y$

$And$

$x^2-y^2 = (x+y)(x-y)$

$Common\ Factor = x+y$

$Non-common\ factor = x-y$

$L.C.M = common\ factor \times non-common\ factor$

$L.C.M = (x+y)(x-y)$

$L.C.M = x^2-y^2$

**(ii)** $x^3-y^3, x-y$

**Solution:**

$x^3-y^3, x-y$

$x^3-y^3 = (x-y)(x^2+xy+y^2)$

And

$x-y = x-y$

$Common\ Factor = x-y$

$Non-common\ factor = x^2+xy+y^2$

$L.C.M = common\ factor \times non-common\ factor$

$L.C.M = (x-y)(x^2+xy+y^2)$

$L.C.M = x^3-y^3$

## Ex # 6.1

**(iii)** $x^5-x, x^5-x^2\ and\ x^5-x^3$

**Solution:**

$x^5-x, x^5-x^2\ and\ x^5-x^3$

$x^5-x = x(x^4-1)$

$= x[(x^2)^2-(1)^1]$

$= x(x^2+1)(x^2-1)$

$= x(x^2+1)(x+1)(x-1)$

And

$x^5-x^2 = x^2(x^3-1)$

$= x.x[(x)^3-(1)^3]$

$= x.x(x-1)(x^2+(x)(1)+1^2)$

$= x.x(x-1)(x^2+x+1)$

And

$x^5-x^3 = x^3(x^2-1)$

$= x.x.x[(x)^2-(1)^2]$

$= x.x.x(x+1)(x-1)$

$Common\ Factor = x(x-1)$

$Non-common\ factor = x.x(x^2+1)(x+1)(x^2+x+1)$

$L.C.M = common\ factor \times non-common\ factor$

$L.C.M = x(x-1) \times x.x(x^2+1)(x+1)(x^2+x+1)$

$L.C.M = x^3(x-1)(x+1)(x^2+1)(x^2+x+1)$

**(iv)** $2^3 3^2(x-y)^3(x+2y)^2, 2^3 3^2(x-y)^2(x+2y)^3, 3^2(x-y)^2(x+2y)$

**Solution:**

$2^3 3^2(x-y)^3(x+2y)^2, 2^3 3^2(x-y)^2(x+2y)^3, 3^2(x-y)^2(x+2y)$

$2^3 3^2(x-y)^3(x+2y)^2 = 2.2.2.3.3(x-y)(x-y)(x-y)(x+2y)(x+2y)$

$2^3 3^2(x-y)^2(x+2y)^3 = 2.2.2.3.3(x-y)(x-y)(x+2y)(x+2y)(x+2y)$

$3^2(x-y)^2(x+2y) = 3.3(x-y)(x-y)(x+2y)$

$Common\ Factor = 3.3(x-y)(x-y)(x+2y)$

$Non-common\ factor = 2.2.2.(x-y)(x+2y)(x+2y)$

$L.C.M = common\ factor \times non-common\ factor$

$L.C.M = 3.3(x-y)(x-y)(x+2y) \times 2.2.2.(x-y)(x+2y)(x+2y)$

$L.C.M = 2^3 3^2(x-y)^3(x+2y)^3$

# Chapter # 6

## Ex # 6.1

**Q4: Find H.C.F and L.C.M of the following**
**160 expression.**

**(i)** $\boldsymbol{x^3 - 2x^2 - 13x - 10}$ **and** $\boldsymbol{x^3 - x^2 - 10x - 8}$

**Solution:**

$x^3 - 2x^2 - 13x - 10$ and $x^3 - x^2 - 10x - 8$

$Let\ A = x^3 - 2x^2 - 13x - 10$

$and\ B = x^3 - x^2 - 10x - 8$

As we have:

$$L.C.M = \frac{A \times B}{H.C.F} - -equ\ (i)$$

First we find H.C.F

$$x^3 - x^2 - 10x - 8 \,\big)\, x^3 - 2x^2 - 13x - 10 \,\big(\, 1$$
$$\pm x^3 \mp x^2 \mp 10x \mp 8$$
$$-1 \,\big)\, -x^2 - 3x - 2$$
$$x^2 + 3x + 2 \,\big)\, x^3 - x^2 - 10x - 8 \,\big(\, x - 4$$
$$\pm x^3 \pm 3x^2 \pm 2x$$
$$-4x^2 - 12x - 8$$
$$\mp 4x^2 \mp 12x \mp 8$$
$$\times$$

$H.C.F = x^2 + 3x + 2$

Now put the values in equ (i)

$$L.C.M = \frac{(x^3 - 2x^2 - 13x - 10)(x^3 - x^2 - 10x - 8)}{x^2 + 3x + 2}$$

Now by Simple Division

$$x - 5$$
$$x^2 + 3x + 2 \,\big)\, x^3 - 2x^2 - 13x - 10$$
$$\pm x^3 \pm 3x^2 \pm 2x$$
$$-5x^2 - 15x - 10$$
$$\mp 5x^2 \mp 15x \mp 10$$
$$\times$$

$So\ L.C.M = (x - 5)(x^3 - x^2 - 10x - 8)$

# Chapter # 6

**Ex # 6.1**

**(ii)** $\mathbf{2x^4-2x^3+x^2+3x-6}$ **and** $\mathbf{4x^4-2x^3+3x-9}$

**Solution:**

$2x^4-2x^3+x^2+3x-6\ and\ 4x^4-2x^3+3x-9$

$Let\ A=2x^4-2x^3+x^2+3x-6$

$and\ B=4x^4-2x^3+3x-9$

As we have:

$$L.C.M=\frac{A\times B}{H.C.F}--equ\ (i)$$

First we find H.C.F

$$2x^4-2x^3+x^2+3x-6\ \overline{)\,4x^4-2x^3+3x-9\,}\ \ 2$$
$$\pm4x^4\mp4x^3\pm6x\mp12\pm2x^2$$
$$2x^3-2x^2-3x+3\ \overline{)\,2x^4-2x^3+x^2+3x-6\,}\ \ x$$
$$\pm2x^4\mp2x^3\mp3x^2\pm3x$$
$$2\ \big|\ 4x^2-6$$
$$2x^2-3\ \overline{)\,2x^3-2x^2-3x+3\,}\ \ x-1$$
$$\pm2x^3\qquad\mp3x$$
$$-2x^2+3$$
$$\mp2x^2\pm3$$
$$\times$$



$H.C.F=2x^2-3$

Now put the values in equ (i)

$$L.C.M=\frac{(2x^4-2x^3+x^2+3x-6)(4x^4-2x^3+3x-9)}{2x^2-3}$$

Now by Simple Division

$$\begin{array}{r|l} & x^2-x+2 \\ \hline 2x^2-3 & 2x^4-2x^3+x^2+3x-6 \\ & \underline{\pm2x^4\qquad\mp3x^2} \\ & -2x^3+4x^2+3x-6 \\ & \underline{\mp2x^3\qquad\pm3x} \\ & 4x^2-6 \\ & \underline{\pm4x^2\mp6} \\ & \times \end{array}$$

$So\ L.C.M=(x^2-x+2)(4x^4-2x^3+3x-9)$

# Chapter # 6

## Ex # 6.1

**(iii)** $\boldsymbol{a^4 - a^3 - a + 1}$ **and** $\boldsymbol{a^4 + a^2 + 1}$

**Solution:**

$a^4 - a^3 - a + 1$ and $a^4 + a^2 + 1$

$Let\ A = a^4 - a^3 - a + 1$

$and\ B = a^4 + a^2 + 1$

As we have:

$$L.C.M = \frac{A \times B}{H.C.F} - -equ\ (i)$$

First we find H.C.F

$$a^4 + a^2 + 1 \;\big)\; a^4 - a^3 - a + 1 \;\big(\; 1$$
$$\pm a^4 \qquad \pm 1 \pm a^2$$
$$-a \;\big)\; -a^3 - a^2 - a$$
$$a^2 + a + 1 \;\big)\; a^4 + a^2 + 1 \;\big(\; a^2 - a + 1$$
$$\pm a^4 \pm a^2 \qquad \pm a^3$$
$$-a^3 + 1$$
$$\mp a^3 \qquad \mp a^2 \mp a$$
$$a^2 + a + 1$$
$$\pm a^2 \pm a \pm 1$$
$$\times$$

$H.C.F = a^2 + a + 1$

Now put the values in equ (i)

$$L.C.M = \frac{(a^4 - a^3 - a + 1)(a^4 + a^2 + 1)}{a^2 + a + 1}$$

Now by Simple Division

$$a^2 - 2a + 1$$
$$a^2 + a + 1 \;\big)\; a^4 - a^3 - a + 1$$
$$\pm a^4 \pm a^3 \qquad \pm a^2$$
$$-2a^3 - a^2 - a + 1$$
$$\mp 2a^3 \mp 2a^2 \mp 2a$$
$$a^2 + a + 1$$
$$\pm a^2 \pm a \pm 1$$
$$\times$$

$So\ L.C.M = (a^2 - 2a + 1)(a^4 + a^2 + 1)$

**Ex # 6.1**

**(iv)** $\mathbf{1 - x^2 - x^4 + x^5}$ **and** $\mathbf{1 + 2x + x^2 - x^4 - x^5}$

**Solution:**

$1 - x^2 - x^4 + x^5$ and $1 + 2x + x^2 - x^4 - x^5$

$x^5 - x^4 - x^2 + 1$ and $-x^5 - x^4 + x^2 + 2x + 1$

$Let\ A = x^5 - x^4 - x^2 + 1$

$and\ B = -x^5 - x^4 + x^2 + 2x + 1$

As we have:

$$L.C.M = \frac{A \times B}{H.C.F} - -equ\ (i)$$

First we find H.C.F

$$\begin{array}{r|l|l}
x^5 - x^4 - x^2 + 1 & -x^5 - x^4 + x^2 + 2x + 1 & -1 \\
 & \mp x^5 \pm x^4 \pm x^2 \qquad \mp 1 & \\
\hline
-2 & -2x^4 + 2x + 2 & \\
\hline
x^4 - x - 1 & x^5 - x^4 - x^2 + 1 & x - 1 \\
 & \pm x^5 \qquad \mp x^2 \qquad \mp x & \\
\hline
 & -x^4 + x + 1 & \\
 & \mp x^4 \pm x \pm 1 & \\
\hline
 & \times &
\end{array}$$

$H.C.F = \quad x^4 - x - 1$

Now put the values in equ (i)

$$L.C.M = \frac{(x^5 - x^4 - x^2 + 1)(-x^5 - x^4 + x^2 + 2x + 1)}{x^4 - x - 1}$$

Now by Simple Division

$$\begin{array}{r|l}
 & \qquad x - 1 \\
\hline
x^4 - x - 1 & x^5 - x^4 - x^2 + 1 \\
 & \pm x^5 \qquad \mp x^2 \qquad \mp x \\
\hline
 & -x^4 + x + 1 \\
 & \mp x^4 \pm x \pm 1 \\
\hline
 & \times
\end{array}$$

$So\ L.C.M = (x + 2)(-x^5 - x^4 + x^2 + 2x + 1)$

$So\ L.C.M = (x + 2)(1 + 2x + x^2 - x^4 - x^5)$

# Chapter # 6

**Q5: 160**

**H.C.F and L.C.M of two polynomials are $x-2$ and $x^3+3x^2-6x-8$ respectively. If one polynomial is $x^2+2x-8$, find the second polynomial.**

Solution:

$H.C.F = x-2$

$L.C.M = x^3+3x^2-6x-8$

$First\ polynomial = A = x^2+2x-8$

$Second\ polynomial = B = ?$

As we have:

$$L.C.M = \frac{A \times B}{H.C.F}$$

$$L.C.M \times H.C.F = A \times B$$

$$\frac{L.C.M \times H.C.F}{A} = B$$

$$B = \frac{L.C.M \times H.C.F}{A}$$

Put the values

$$B = \frac{(x^3+3x^2-6x-8)(x-2)}{x^2+2x-8}$$

Now by simple Division

$$\begin{array}{r|l} & x+1 \\ \hline x^2+2x-8, & x^3+3x^2-6x-8 \\ & \pm x^3 \pm 2x^2 \mp 8x \\ \hline & x^2+2x-8 \\ & \pm x^2 \pm 2x \mp 8 \\ \hline & \times \end{array}$$

$So\ B = (x+1)(x-2)$

$B = x^2-2x+1x-2$

$B = x^2-x-2$

**Q6: 160**

**If product of two polynomials is $x^4+5x^3-6x^2-2x-28$ and their H.C.F is $x-2$. Find their L.C.M.**

Solution:

Let Product of two polynomials $= A \times B$

$Then\ A \times B = x^4+5x^3-6x^2-2x-28$

$H.C.F = x-2$

$L.C.M = ?$

As we have:

$$L.C.M = \frac{A \times B}{H.C.F}$$

Put the values

$$L.C.M = \frac{x^4+5x^3-6x^2-2x-28}{x-2}$$

$$\begin{array}{r|l} & x^3+7x^2+8x+14 \\ \hline x-2 & x^4+5x^3-6x^2-2x-28 \\ & \pm x^4 \mp 2x^3 \\ \hline & 7x^3-6x^2-2x-28 \\ & \pm 7x^3 \mp 14x^2 \\ \hline & 8x^2-2x-28 \\ & \pm 8x^2 \mp 16x \\ \hline & 14x-28 \\ & \pm 14 \mp 28 \\ \hline & \times \end{array}$$

$L.C.M = x^3+7x^2+8x+14$

**Q7: 160**

**H.C.F and L.C.M of two polynomials are $x+5$ and $2x^3+11x^2+2x-15$ respectively. Find the polynomials of degree 2.**

**Solution:**

$H.C.F = x+5$

$L.C.M = 2x^3+11x^2+2x-15$

$First\ polynomial = A = ?$

$Second\ polynomial = B = ?$

$As\ H.C.F = x+5$

$then\ it\ is\ also\ the\ factor\ of\ L.C.M$

Now

$$\begin{array}{r|l} & 2x^2+x-3 \\ \hline x+5 & 2x^3+11x^2+2x-15 \\ & \pm 2x^3 \pm 10x^2 \\ \hline & x^2+2x-15 \\ & \pm x^2 \pm 5x \\ \hline & -3x-15 \\ & \mp 3x \mp 15 \\ \hline & \times \end{array}$$

$L.C.M = 2x^3+11x^2+2x-15$

$L.C.M = (x+5)(2x^2+x-3)$

$L.C.M = (x+5)(2x^2+3x-2x-3)$

$L.C.M = (x+5)[x(2x+3)-1(2x+3)]$

$L.C.M = (x+5)(2x+3)(x-1)$

$As\ x+5\ is\ common\ factor. So$

$A = (x+5)(2x+3)$

$A = 2x^2+3x+10x+15$

$A = 2x^2+13x+15$

And

$B = (x+5)(x-1)$

$B = x^2-1x+5x-5$

$B = x^2+4x-5$

# Chapter # 6

## Ex # 6.1

**Q8: 160** **If product of two polynomials is $x^4 + 6x^3 - 3x^2 - 56x - 48$ and their L.C.M is $x^3 + 2x^2 - 11x - 12$. Find their H.C.F.**

**Solution:**

Let Product of two polynomials $= A \times B$

$Then\ A \times B = x^4 + 6x^3 - 3x^2 - 56x - 48$

$L.C.M = x^3 + 2x^2 - 11x - 12$

$H.C.F = ?$

As we have:

$$L.C.M = \frac{A \times B}{H.C.F}$$

$$H.C.F = \frac{A \times B}{L.C.M}$$

Put the values

$$H.C.F = \frac{x^4 + 6x^3 - 3x^2 - 56x - 48}{x^3 + 2x^2 - 11x - 12}$$

Now by Simple Division

$$
\begin{array}{r|l}
 & x + 4 \\
x^3 + 2x^2 - 11x - 12 & x^4 + 6x^3 - 3x^2 - 56x - 48 \\
 & \pm x^4 \pm 2x^3 \mp 11x^2 \mp 12x \\
\hline
 & 4x^3 + 8x^2 - 44x - 48 \\
 & \pm 4x^3 \pm 8x^2 \mp 44x \mp 48 \\
\hline
 & \times
\end{array}
$$

So $H.C.F = x + 4$

**Q9: 160** **Waqar wishes to distribute 128 bananas and also 176 apples equally among certain number of children. Find the highest number of children who can get the fruit in this way.**

Solution:

Bananas = 128

Apples = 176

Highest number of children = ?

Now

| | |
|---|---|
| 2 | 128 |
| 2 | 64 |
| 2 | 32 |
| 2 | 16 |
| 2 | 8 |
| 2 | 4 |
| 2 | 2 |
| | 1 |

| | |
|---|---|
| 2 | 176 |
| 2 | 88 |
| 2 | 44 |
| 2 | 22 |
| 11 | 11 |
| | 1 |

$128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$176 = 2 \times 2 \times 2 \times 2 \times 11$

$H.C.F = 2 \times 2 \times 2 \times 2$

$= 16$

So highest number of children = 16

## Ex # 6.2

**Algebraic fractions**

An algebraic fraction is the quotient of two algebraic expressions.

**Example:**

$$\frac{x - y}{y^2 - 4x^2}$$

### Example # 12

**Simplify** $\boldsymbol{\frac{x + y}{3x + 2y} + \frac{x - y}{3x + 2y}}$

**Solution:**

$$\frac{x + y}{3x + 2y} + \frac{x - y}{3x + 2y}$$

$$= \frac{x + y + x - y}{3x + 2y}$$

$$= \frac{x + x + y - y}{3x + 2y}$$

$$= \frac{2x}{3x + 2y}$$

### Example # 13

**Simplify** $\boldsymbol{\frac{x - y}{x + y} - \frac{x^2 - 2y^2}{x^2 - y^2}}$

**Solution:**

$$\frac{x - y}{x + y} - \frac{x^2 - 2y^2}{x^2 - y^2}$$

$$= \frac{x - y}{x + y} - \frac{x^2 - 2y^2}{(x + y)(x - y)}$$

$$= \frac{(x - y)(x - y) - (x^2 - 2y^2)}{(x + y)(x - y)}$$

$$= \frac{(x - y)^2 - x^2 + 2y^2}{(x + y)(x - y)}$$

$$= \frac{x^2 + y^2 - 2xy - x^2 + 2y^2}{(x + y)(x - y)}$$

$$= \frac{x^2 - x^2 + 2y^2 + y^2 - 2xy}{(x + y)(x - y)}$$

$$= \frac{3y^2 - 2xy}{x^2 - y^2}$$

# Chapter # 6

## Ex # 6.2

**Example # 14**

**Simplify** $\dfrac{x^2-xy+y^2}{x^3+y^3}+\dfrac{x^2+xy+y^2}{x^3-y^3}-\dfrac{1}{x^2-y^2}$

**Solution:**

$$\frac{x^2-xy+y^2}{x^3+y^3}+\frac{x^2+xy+y^2}{x^3-y^3}-\frac{1}{x^2-y^2}$$

$$=\frac{x^2-xy+y^2}{(x+y)(x^2-xy+y^2)}+\frac{x^2+xy+y^2}{(x-y)(x^2+xy+y^2)}-\frac{1}{(x+y)(x-y)}$$

$$=\frac{1}{x+y}+\frac{1}{x-y}-\frac{1}{(x+y)(x-y)}$$

$$=\frac{1(x-y)+1(x+y)-1}{(x+y)(x-y)}$$

$$=\frac{x-y+x+y-1}{(x+y)(x-y)}$$

$$=\frac{x+x-y+y-1}{x^2-y^2}$$

$$=\frac{2x-1}{x^2-y^2}$$

**Example # 15**

**Simplify** $\dfrac{y}{y^2-y-2}-\dfrac{1}{y^2+5y-14}-\dfrac{2}{y^2+8y+7}$

**Solution:**

$$\frac{y}{y^2-y-2}-\frac{1}{y^2+5y-14}-\frac{2}{y^2+8y+7}$$

$$=\frac{y}{y^2-2y+y-2}-\frac{1}{y^2-2y+7y-14}-\frac{2}{y^2+1y+7y+7}$$

$$=\frac{y}{y(y-2)+1(y-2)}-\frac{1}{y(y-2)+7(y-2)}-\frac{2}{y(y+1)+7(y+1)}$$

$$=\frac{y}{(y-2)(y+1)}-\frac{1}{(y-2)(y+7)}-\frac{2}{(y+1)(y+7)}$$

$$=\frac{y(y+7)-1(y+1)-2(y-2)}{(y-2)(y+1)(y+7)}$$

$$=\frac{y^2+7y-1y-1-2y+4}{(y-2)(y+1)(y+7)}$$

$$=\frac{y^2+6y-2y-1+4}{(y-2)(y+1)(y+7)}$$

$$=\frac{y^2+4y+3}{(y-2)(y+1)(y+7)}$$

$$=\frac{y^2+1y+3y+3}{(y-2)(y+1)(y+7)}$$

$$=\frac{y(y+1)+3(y+1)}{(y-2)(y+1)(y+7)}$$

$$=\frac{(y+1)(y+3)}{(y-2)(y+1)(y+7)}$$

$$=\frac{y+3}{(y-2)(y+7)}$$

## Ex # 6.2

**Example # 16**

***Simplify*** $\dfrac{x+4}{x-3}\times\dfrac{x^2-9}{x^2-x-2}$

**Solution:**

$$\frac{x+4}{x-3}\times\frac{x^2-9}{x^2-x-2}$$

$$=\frac{x+4}{x-3}\times\frac{x^2-3^2}{x^2-2x+1x-2}$$

$$=\frac{x+4}{x-3}\times\frac{(x+3)(x-3)}{x(x-2)+1(x-2)}$$

$$=\frac{x+4}{x-3}\times\frac{(x+3)(x-3)}{(x-2)(x+1)}$$

$$=\frac{x+4}{1}\times\frac{(x+3)}{(x-2)(x+1)}$$

$$=\frac{(x+4)(x+3)}{(x-2)(x+1)}$$

**Example # 17**

**Multiply** $\dfrac{x^2-2x}{2x^2+5x+3}$ **by** $\dfrac{2x^2-3x-9}{x^2-9}$

**Solution:**

$$\frac{x^2-2x}{2x^2+5x+3}\times\frac{2x^2-3x-9}{x^2-9}$$

$$=\frac{x(x-2)}{2x^2+2x+3x+3}\times\frac{2x^2+3x-6x-9}{x^2-9^2}$$

$$=\frac{x(x-2)}{2x(x+1)+3(x+1)}\times\frac{x(2x+3)-3(2x+3)}{(x+3)(x-3)}$$

$$=\frac{x(x-2)}{(x+1)(2x+3)}\times\frac{(2x+3)(x-3)}{(x+3)(x-3)}$$

$$=\frac{x(x-2)}{(x+1)}\times\frac{1}{(x+3)}$$

$$=\frac{x(x-2)}{(x+1)(x+3)}$$

**Example # 18**

$$\left(\frac{x^3-y^3}{y^3}\times\frac{y}{x-y}\right)\div\frac{x^2+xy+y^2}{y^2}$$

**Solution:**

$$\left(\frac{x^3-y^3}{y^3}\times\frac{y}{x-y}\right)\div\frac{x^2+xy+y^2}{y^2}$$

$$=\frac{x^3-y^3}{y^3}\times\frac{y}{x-y}\times\frac{y^2}{x^2+xy+y^2}$$

$$=\frac{(x-y)(x^2+xy+y^2)}{y.y.y}\times\frac{y}{x-y}\times\frac{y.y}{x^2+xy+y^2}$$

$$=1$$

# Chapter # 6

## Ex # 6.2

**Q1: Simplify:**

$(i)\ \dfrac{x}{x+y}+\dfrac{2y}{x+y}$

**Solution:**

$$\frac{x}{x+y}+\frac{2y}{x+y}$$
$$=\frac{x+2y}{x+y}$$

$(ii)\ \dfrac{x+y}{3x+2y}+\dfrac{x-y}{3x+2y}$

**Solution:**

$$\frac{x+y}{3x+2y}+\frac{x-y}{3x+2y}$$
$$=\frac{x+y+x-y}{3x+2y}$$
$$=\frac{x+x+y-y}{3x+2y}$$
$$=\frac{2x}{3x+2y}$$

$(iii)\ \dfrac{3}{y-2}-\dfrac{2}{y+2}-\dfrac{y}{y^2-4}$

**Solution:**

$$\frac{3}{y-2}-\frac{2}{y+2}-\frac{y}{y^2-4}$$
$$=\frac{3}{y-2}-\frac{2}{y+2}-\frac{y}{y^2-(2)^2}$$
$$=\frac{3}{y-2}-\frac{2}{y+2}-\frac{y}{(y+2)(y-2)}$$
$$=\frac{3(y+2)-2(y-2)-y}{(y+2)(y-2)}$$
$$=\frac{3y+6-2y+4-y}{(y+2)(y-2)}$$
$$=\frac{3y-2y-y+6+4}{(y+2)(y-2)}$$
$$=\frac{3y-3y+10}{y^2-(2)^2}$$
$$=\frac{10}{y^2-4}$$

$(iv)\ \dfrac{x-y}{x+y}-\dfrac{x^2-2y^2}{x^2-y^2}$

**Solution:**

$$\frac{x-y}{x+y}-\frac{x^2-2y^2}{x^2-y^2}$$

## Ex # 6.2

$$=\frac{x-y}{x+y}-\frac{x^2-2y^2}{(x+y)(x-y)}$$
$$=\frac{(x-y)(x-y)-(x^2-2y^2)}{(x+y)(x-y)}$$
$$=\frac{(x-y)^2-x^2+2y^2}{(x+y)(x-y)}$$
$$=\frac{x^2+y^2-2xy-x^2+2y^2}{(x+y)(x-y)}$$
$$=\frac{x^2-x^2+2y^2+y^2-2xy}{(x+y)(x-y)}$$
$$=\frac{3y^2-2xy}{x^2-y^2}$$

$(v)\ \dfrac{x}{2x^2+3xy+y^2}-\dfrac{x-y}{y^2-4x^2}+\dfrac{y}{2x^2+xy-y^2}$

**Solution:**

$$\frac{x}{2x^2+3xy+y^2}-\frac{x-y}{y^2-4x^2}+\frac{y}{2x^2+xy-y^2}$$
$$=\frac{x}{2x^2+2xy+1xy+y^2}-\frac{x-y}{-4x^2+y^2}+\frac{y}{2x^2+2xy-1xy-y^2}$$
$$=\frac{x}{2x(x+y)+y(x+y)}-\frac{x-y}{-(4x^2-y^2)}+\frac{y}{2x(x+y)-y(x+y)}$$
$$=\frac{x}{(x+y)(2x+y)}+\frac{x-y}{(2x)^2-y^2}+\frac{y}{(x+y)(2x-y)}$$
$$=\frac{x}{(x+y)(2x+y)}+\frac{x-y}{(2x+y)(2x-y)}+\frac{y}{(x+y)(2x-y)}$$
$$=\frac{x(2x-y)+(x-y)(x+y)+y(2x+y)}{(x+y)(2x+y)(2x-y)}$$
$$=\frac{2x^2-xy+x^2-y^2+2xy+y^2}{(x+y)(2x+y)(2x-y)}$$
$$=\frac{2x^2+x^2-xy+2xy-y^2+y^2}{(x+y)((2x)^2-y^2)}$$
$$=\frac{3x^2+xy}{(x+y)(4x^2-y^2)}$$

$(vi)\ \dfrac{a}{3x-y}+\dfrac{a}{3x+y}-\dfrac{6ax}{9x^2-y^2}$

**Solution:**

$$\frac{a}{3x-y}+\frac{a}{3x+y}-\frac{6ax}{9x^2-y^2}$$
$$=\frac{a}{3x-y}+\frac{a}{3x+y}-\frac{6ax}{(3x)^2-y^2}$$

# Chapter # 6

**Ex # 6.2**

$$= \frac{a}{3x-y} + \frac{a}{3x+y} - \frac{6ax}{(3x+y)(3x-y)}$$

$$= \frac{a(3x+y) + a(3x-y) - 6ax}{(3x+y)(3x-y)}$$

$$= \frac{3ax + ay + 3ax - ay - 6ax}{(3x+y)(3x-y)}$$

$$= \frac{3ax + 3ax - 6ax + ay - ay}{(3x+y)(3x-y)}$$

$$= \frac{6ax - 6ax}{(3x+y)(3x-y)}$$

$$= \frac{0}{(3x+y)(3x-y)}$$

$$= 0$$

---

$(\boldsymbol{vii})$ $\boldsymbol{\dfrac{y}{x-y} + \dfrac{y}{x+y} + \dfrac{2xy}{x^2+y^2} + \dfrac{4x^3y}{x^4+y^4}}$

**Solution:**

$$\frac{y}{x-y} + \frac{y}{x+y} + \frac{2xy}{x^2+y^2} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{y(x+y) + y(x-y)}{(x-y)(x+y)} + \frac{2xy}{x^2+y^2} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{xy + y^2 + xy - y^2}{(x-y)(x+y)} + \frac{2xy}{x^2+y^2} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{xy + xy + y^2 - y^2}{x^2-y^2} + \frac{2xy}{x^2+y^2} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{2xy}{x^2-y^2} + \frac{2xy}{x^2+y^2} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{2xy(x^2+y^2) + 2xy(x^2-y^2)}{(x^2-y^2)(x^2+y^2)} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{2x^3y + 2xy^3 + 2x^3y - 2xy^3}{(x^2)^2 - (y^2)^2} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{2x^3y + 2x^3y + 2xy^3 - 2xy^3}{x^4-y^4} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{4x^3y}{x^4-y^4} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{4x^3y(x^4+y^4) + 4x^3y(x^4-y^4)}{(x^4-y^4)(x^4+y^4)}$$

**Ex # 6.2**

$$= \frac{4x^7y + 4x^3y^5 + 4x^7y - 4x^3y^5}{(x^4)^2 - (y^4)^2}$$

$$= \frac{4x^7y + 4x^7y + 4x^3y^5 - 4x^3y^5}{x^8 - y^8}$$

$$= \frac{8x^7y}{x^8-y^8}$$

$(\boldsymbol{viii})$ $\boldsymbol{\dfrac{1}{a^2+7a+10} + \dfrac{1}{a^2+10a+16}}$

**Solution:**

$$\frac{1}{a^2+7a+10} + \frac{1}{a^2+10a+16}$$

$$= \frac{1}{a^2+2a+5a+10} + \frac{1}{a^2+2a+8a+16}$$

$$= \frac{1}{a(a+2)+5(a+2)} + \frac{1}{a(a+2)+8(a+2)}$$

$$= \frac{1}{(a+2)(a+5)} + \frac{1}{(a+2)(a+8)}$$

$$= \frac{1(a+8) + 1(a+5)}{(a+2)(a+5)(a+8)}$$

$$= \frac{a+8+a+5}{(a+2)(a+5)(a+8)}$$

$$= \frac{a+a+8+5}{(a+2)(a+5)(a+8)}$$

$$= \frac{2a+13}{(a+2)(a+5)(a+8)}$$

---

$(\boldsymbol{ix})$ $\boldsymbol{\dfrac{1}{a-b} + \dfrac{1}{a+b} + \dfrac{2a}{a^2+b^2} + \dfrac{4a^3}{a^4+b^4}}$

**Solution:**

$$\frac{1}{a-b} + \frac{1}{a+b} + \frac{2a}{a^2+b^2} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{1(a+b) + 1(a-b)}{(a-b)(a+b)} + \frac{2a}{a^2+b^2} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{a+b+a-b}{(a-b)(a+b)} + \frac{2a}{a^2+b^2} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{a+a+b-b}{a^2-b^2} + \frac{2a}{a^2+b^2} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{2a}{a^2-b^2} + \frac{2a}{a^2+b^2} + \frac{4a^3}{a^4+b^4}$$

# Chapter # 6

## Ex # 6.2

$$= \frac{2a(a^2+b^2)+2a(a^2-b^2)}{(a^2-b^2)(a^2+b^2)} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{2a^3+2ab^2+2a^3-2ab^2}{(a^2)^2-(b^2)^2} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{2a^3+2a^3+2ab^2-2ab^2}{a^4-b^4} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{4a^3}{a^4-b^4} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{4a^3(a^4+b^4)+4a^3(a^4-b^4)}{(a^4-b^4)(a^4+b^4)}$$

$$= \frac{4a^7+4a^3b^4+4a^7-4a^3b^4}{(a^4)^2-(b^4)^2}$$

$$= \frac{4a^7+4a^7+4a^3b^4-4a^3b^4}{a^8-b^8}$$

$$= \frac{8a^7}{a^8-b^8}$$

(x) $\boldsymbol{\frac{x^2-xy+y^2}{x^3+y^3} + \frac{x^2+xy+y^2}{x^3-y^3} - \frac{1}{x^2-y^2}}$

**Solution:**

$$\frac{x^2-xy+y^2}{x^3+y^3} + \frac{x^2+xy+y^2}{x^3-y^3} - \frac{1}{x^2-y^2}$$

$$= \frac{x^2-xy+y^2}{(x+y)(x^2-xy+y^2)} + \frac{x^2+xy+y^2}{(x-y)(x^2+xy+y^2)} - \frac{1}{(x+y)(x-y)}$$

$$= \frac{1}{x+y} + \frac{1}{x-y} - \frac{1}{(x+y)(x-y)}$$

$$= \frac{1(x-y)+1(x+y)-1}{(x+y)(x-y)}$$

$$= \frac{x-y+x+y-1}{(x+y)(x-y)}$$

$$= \frac{x+x-y+y-1}{x^2-y^2}$$

$$= \frac{2x-1}{x^2-y^2}$$

## Ex # 6.2

**Q2: Simplify**

(i) $\boldsymbol{\frac{x^2-25}{5-x}}$

**Solution:**

$$\frac{x^2-25}{5-x}$$

$$= \frac{x^2-(5)^2}{-x+5}$$

$$= \frac{(x+5)(x-5)}{-(x-5)}$$

$$= -(x+5)$$

(ii) $\boldsymbol{\frac{x^2+5x+4}{4y^3} \times \frac{2y^2}{x^2+3x+2}}$

**Solution:**

$$\frac{x^2+5x+4}{4y^3} \times \frac{2y^2}{x^2+3x+2}$$

$$= \frac{x^2+4x+1x+4}{4y.y.y} \times \frac{2y.y}{x^2+2x+1x+2}$$

$$= \frac{x(x+4)+1(x+4)}{2y} \times \frac{1}{x(x+2)+1(x+2)}$$

$$= \frac{(x+4)(x+1)}{2y} \times \frac{1}{(x+2)(x+1)}$$

$$= \frac{x+4}{2y} \times \frac{1}{x+2}$$

$$= \frac{x+4}{2y(x+2)}$$

(iii) $\boldsymbol{\frac{x^2-5x+4}{x^3-3x-4} \div \frac{x^3-4x^2+x-4}{2x-1}}$

**Solution:**

$$\frac{x^2-5x+4}{x^2-3x-4} \div \frac{x^3-4x^2+x-4}{2x-1}$$

$$= \frac{x^2-5x+4}{x^2-3x-4} \times \frac{2x-1}{x^3-4x^2+x-4}$$

$$= \frac{x^2-4x-1x+4}{x^2-4x+1x-4} \times \frac{2x-1}{x^3-4x^2+x-4}$$

$$= \frac{x(x-4)-1(x-4)}{x(x-4)+1(x-4)} \times \frac{2x-1}{x^2(x-4)+1(x-4)}$$

$$= \frac{(x-4)(x-1)}{(x-4)(x+1)} \times \frac{2x-1}{(x-4)(x^2+1)}$$

# Chapter # 6

**Ex # 6.2**

$$=\frac{(x-1)}{(x+1)}\times\frac{2x-1}{(x-4)(x^2+1)}$$

$$=\frac{(x-1)(2x-1)}{(x+1)(x-4)(x^2+1)}$$

---

$(iv)$ $\boldsymbol{\frac{a(a+b)}{a^3-b^3}\times\frac{a^2+ab+b^2}{a^2+b^2}}$

**Solution:**

$$\frac{a(a+b)}{a^3-b^3}\times\frac{a^2+ab+b^2}{a^2+b^2}$$

$$=\frac{a(a+b)}{(a-b)(a^2+ab+b^2)}\times\frac{a^2+ab+b^2}{a^2+b^2}$$

$$=\frac{a(a+b)}{(a-b)}\times\frac{1}{a^2+b^2}$$

$$=\frac{a(a+b)}{(a-b)(a^2+b^2)}$$

---

$(v)$ $\boldsymbol{\frac{7}{x^2-4}\div\frac{xy}{x+2}}$

**Solution:**

$$\frac{7}{x^2-4}\div\frac{xy}{x+2}$$

$$=\frac{7}{x^2-2^2}\times\frac{x+2}{xy}$$

$$=\frac{7}{(x+2)(x-2)}\times\frac{x+2}{xy}$$

$$=\frac{7}{x-2}\times\frac{1}{xy}$$

$$=\frac{7}{xy(x-2)}$$

---

$(vi)$ $\boldsymbol{\frac{a^3-b^3}{a^4-b^4}\div\frac{a^2+ab+b^2}{a^2+b^2}}$

**Solution:**

$$\frac{a^3-b^3}{a^4-b^4}\div\frac{a^2+ab+b^2}{a^2+b^2}$$

$$=\frac{a^3-b^3}{a^4-b^4}\times\frac{a^2+b^2}{a^2+ab+b^2}$$

$$=\frac{(a-b)(a^2+ab+b^2)}{(a^2+b^2)(a^2-b^2)}\times\frac{a^2+b^2}{a^2+ab+b^2}$$

**Ex # 6.2**

$$=\frac{(a-b)(a^2+ab+b^2)}{(a^2+b^2)(a+b)(a-b)}\times\frac{a^2+b^2}{a^2+ab+b^2}$$

$$=\frac{1}{(a+b)}\times\frac{1}{1}$$

$$=\frac{1}{(a+b)}$$

---

$(vii)$ $\boldsymbol{\frac{2x}{3x-12}\div\frac{x^2-2x}{x^2-6x+8}}$

**Solution:**

$$\frac{2x}{3x-12}\div\frac{x^2-2x}{x^2-6x+8}$$

$$=\frac{2x}{3x-12}\times\frac{x^2-6x+8}{x^2-2x}$$

$$=\frac{2x}{3(x-4)}\times\frac{x^2-2x-4x+8}{x(x-2)}$$

$$=\frac{2x}{3(x-4)}\times\frac{x(x-2)-4(x-2)}{x(x-2)}$$

$$=\frac{2x}{3(x-4)}\times\frac{(x-2)(x-4)}{x(x-2)}$$

$$=\frac{2}{3}\times\frac{1}{1}$$

$$=\frac{2}{3}$$

---

$(viii)$ $\boldsymbol{\frac{a^4-8a}{2a^2+5a-3}\times\frac{2a-1}{a^2+2a+4}\div\frac{a^2-2a}{a+3}}$

**Solution:**

$$\frac{a^4-8a}{2a^2+5a-3}\times\frac{2a-1}{a^2+2a+4}\div\frac{a^2-2a}{a+3}$$

$$=\frac{a^4-8a}{2a^2+5a-3}\times\frac{2a-1}{a^2+2a+4}\times\frac{a+3}{a^2-2a}$$

$$=\frac{a(a^3-8)}{2a^2+6a-1a-3}\times\frac{2a-1}{a^2+2a+4}\times\frac{a+3}{a(a-2)}$$

$$=\frac{a(a^3-2^3)}{2a(a+3)-1(a+3)}\times\frac{2a-1}{a^2+2a+4}\times\frac{a+3}{a(a-2)}$$

$$=\frac{a(a-2)(a^2+2a+4)}{(a+3)(2a-1)}\times\frac{2a-1}{a^2+2a+4}\times\frac{a+3}{a(a-2)}$$

$$=1$$

# Chapter # 6

**Ex # 6.2**

(ix) $\dfrac{9-x^2}{x^4+6x^3} \div \dfrac{x^3-2x^2-3x}{x^2+7x+6}$

**Solution:**

$$\frac{9-x^2}{x^4+6x^3} \div \frac{x^3-2x^2-3x}{x^2+7x+6}$$

$$= \frac{-x^2+9}{x^4+6x^3} \times \frac{x^2+7x+6}{x^3-2x^2-3x}$$

$$= \frac{-(x^2-9)}{x^3(x+6)} \times \frac{x^2+1x+6x+6}{x(x^2-2x-3)}$$

$$= \frac{-(x^2-3^2)}{x^3(x+6)} \times \frac{x(x+1)+6(x+1)}{x(x^2-3x+1x-3)}$$

$$= \frac{-(x+3)(x-3)}{x^3(x+6)} \times \frac{(x+1)(x+6)}{x[x(x-3)+1(x-3)]}$$

$$= \frac{-(x+3)(x-3)}{x^3(x+6)} \times \frac{(x+1)(x+6)}{x[(x-3)(x+1)]}$$

$$= \frac{-(x+3)}{x^3} \times \frac{1}{x}$$

$$= \frac{-(x+3)}{x^4}$$

(x) $\dfrac{ax+ab+cx+bc}{a^2-x^2} \times \dfrac{x^2-2ax+a^2}{x^2+(b+a)x+ab}$

**Solution:**

$$\frac{ax+ab+cx+bc}{a^2-x^2} \times \frac{x^2-2ax+a^2}{x^2+(b+a)x+ab}$$

$$= \frac{ax+ab+cx+bc}{-x^2+a^2} \times \frac{x^2-2ax+a^2}{x^2+bx+ax+ab}$$

$$= \frac{a(x+b)+c(x+b)}{-(x^2-a^2)} \times \frac{(x-a)^2}{x(x+b)+a(x+b)}$$

$$= -\frac{(x+b)(a+c)}{(x+a)(x-a)} \times \frac{(x-a)(x-a)}{(x+b)(x+a)}$$

$$= -\frac{(a+c)}{(x+a)} \times \frac{(x-a)}{(x+a)}$$

$$= -\frac{(a+c)(x-a)}{(x+a)^2}$$

**Ex # 6.3**

**Square root**

Square root of a number is a number that can be multiplied by itself to produce the original

Square root of an algebraic expression can be found out by the following two methods.

(i) Factorization Method

(ii) Division Method

**Square root by Factorization**

In this method make the expression a perfect square then finds square root.

**Example # 20**

***Find the square root of*** $\boldsymbol{x^2+ax+\frac{1}{4}a^2}$ **by factorization**

**Solution:**

$$x^2+ax+\frac{1}{4}a^2$$

$$x^2+ax+\frac{1}{4}a^2 = (x)^2+2(x)\left(\frac{1}{2}a\right)+\left(\frac{1}{2}a\right)^2$$

$$x^2+ax+\frac{1}{4}a^2 = \left(x+\frac{1}{2}a\right)^2$$

Now take square root on B.S

$$\sqrt{x^2+ax+\frac{1}{4}a^2} = \sqrt{\left(x+\frac{1}{2}a\right)^2}$$

$$\sqrt{x^2+ax+\frac{1}{4}a^2} = \pm\left(x+\frac{1}{2}a\right)$$

**Example # 21**

***Find the square root of*** $\boldsymbol{x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27}$

**Solution:**

$$x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27$$

$$x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27 = x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+25+2$$

$$x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27 = x^2+\frac{1}{x^2}+2-10\left(x+\frac{1}{x}\right)+25$$

$$x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27 = \left(x+\frac{1}{x}\right)^2-2\left(x+\frac{1}{x}\right)(5)+(5)^2$$

$$x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27 = \left(x+\frac{1}{x}-5\right)^2$$

**Ex # 6.3**

Taking square root on B.S

$$\sqrt{x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27}=\sqrt{\left(x+\frac{1}{x}-5\right)^2}$$

$$\sqrt{x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27}=\pm\left(x+\frac{1}{x}-5\right)$$

**Square root by Division**

طریقہ:

Expression کو Descending ترتیب میں لکھیں۔

پہلے expression کا Square root لینگے پھر Divisor اور Quotient میں لکھیں گے۔

Divisor اور Quotient کو آپس میں Multiply کریں اور پہلے expression کے نیچے لکھیں پھر Subtract کریں تو Remainder حاصل ہو جائے گا

Divisor کو ڈبل کردے اور Remainder کو اس پر Divide کردے اور جو Term آئے گا تو Divisor اور Quotient میں اس کو لکھیں۔

اب اس Quotient کو پورے Divisor کے ساتھ Multiply کرے پھر Subtract کرے

اب Divisor کے دوسرے Term کو ڈبل کرے اور اوپر کا طریقہ دوبارہ کریں۔

**Find the square root of $16x^4-24x^3+25x^2-12x+4$**

**Solution:**

Write the expression in descending order

$16x^4-24x^3+25x^2-12x+4$

Take the square root of first element of expression.

$$\sqrt{16x^4}=4x^2$$

Write $4x^2$ in divisor and quotient

$$\begin{array}{r|l}
\multicolumn{2}{c}{4x^2} \\ \cline{2-2}
4x^2 & 16x^4-24x^3+25x^2-12x+4
\end{array}$$

Multiply the divisor and quotient and write it under first expression then subtract from given expression to get the remainder.

$$\begin{array}{r|l}
\multicolumn{2}{c}{4x^2} \\ \cline{2-2}
4x^2 & 16x^4-24x^3+25x^2-12x+4 \\
 & \pm16x^4 \\ \cline{2-2}
 & -24x^3+25x^2-12x+4
\end{array}$$

Now twice the divisor

$$\begin{array}{r|l}
\multicolumn{2}{c}{4x^2} \\ \cline{2-2}
4x^2 & 16x^4-24x^3+25x^2-12x+4 \\
 & \pm16x^4 \\ \cline{2-2}
8x^2 & -24x^3+25x^2-12x+4
\end{array}$$

Divide the 2nd expression by this divisor then write that term in quotient and with this divisor.

$$\frac{-24x^3}{8x^2}=-3x$$

$$\begin{array}{r|l}
\multicolumn{2}{c}{4x^2-3x} \\ \cline{2-2}
4x^2 & 16x^4-24x^3+25x^2-12x+4 \\
 & \pm16x^4 \\ \cline{2-2}
8x^2-3x & -24x^3+25x^2-12x+4
\end{array}$$

Multiply this quotient with entire divisor

$$-3x(8x^2-3x)=-24x^3+9x^2$$

Write $-24x^3+9x^2$ under given expression then subtract it.

$$\begin{array}{r|l}
\multicolumn{2}{c}{4x^2-3x} \\ \cline{2-2}
4x^2 & 16x^4-24x^3+25x^2-12x+4 \\
 & \pm16x^4 \\ \cline{2-2}
8x^2-3x & -24x^3+25x^2-12x+4 \\
 & \mp24x^3\pm9x^2 \\ \cline{2-2}
 & 16x^2-12x+4
\end{array}$$

Now twice the 2nd term of the divisor

$$\begin{array}{r|l}
\multicolumn{2}{c}{4x^2-3x} \\ \cline{2-2}
4x^2 & 16x^4-24x^3+25x^2-12x+4 \\
 & \pm16x^4 \\ \cline{2-2}
8x^2-3x & -24x^3+25x^2-12x+4 \\
 & \mp24x^3\pm9x^2 \\ \cline{2-2}
8x^2-6x & 16x^2-12x+4
\end{array}$$

Repeat the above procedure.

Divide $16x^2$ by divisor $8x^2$ then write that term in quotient and with this divisor.

$$\frac{16x^2}{8x^2}=2$$

$$\begin{array}{r|l}
\multicolumn{2}{c}{4x^2-3x+2} \\ \cline{2-2}
4x^2 & 16x^4-24x^3+25x^2-12x+4 \\
 & \pm16x^4 \\ \cline{2-2}
8x^2-3x & -24x^3+25x^2-12x+4 \\
 & \mp24x^3\pm9x^2 \\ \cline{2-2}
8x^2-6x+2 & 16x^2-12x+4
\end{array}$$

Multiply this quotient with entire divisor

$$2(8x^2-6x+2)=16x^2-12x+4$$

Write $16x^2-12x+4$ under given expression then subtract it.

$$\begin{array}{r|l}
\multicolumn{2}{c}{4x^2-3x+2} \\ \cline{2-2}
4x^2 & 16x^4-24x^3+25x^2-12x+4 \\
 & \pm16x^4 \\ \cline{2-2}
8x^2-3x & -24x^3+25x^2-12x+4 \\
 & \mp24x^3\pm9x^2 \\ \cline{2-2}
8x^2-6x+2 & 16x^2-12x+4 \\
 & \pm16x^2\mp12x\pm4 \\ \cline{2-2}
 & 0
\end{array}$$

# Chapter # 6

**Ex # 6.3**

**Example # 22**

**Find the square root of $16x^4 - 24x^3 + 25x^2 - 12x + 4$**

**Solution:**

Now

$$\begin{array}{r|l} & \quad 4x^2 - 3x + 2 \\ \hline 4x^2 & 16x^4 - 24x^3 + 25x^2 - 12x + 4 \\ & \pm 16x^4 \\ \hline 8x^2 - 3x & -24x^3 + 25x^2 - 12x + 4 \\ & \mp 24x^3 \pm 9x^2 \\ \hline 8x^2 - 6x + 2 & 16x^2 - 12x + 4 \\ & \pm 16x^2 \mp 12x \pm 4 \\ \hline & 0 \end{array}$$

So

$$\sqrt{16x^4 - 24x^3 + 25x^2 - 12x + 4} = \pm(4x^2 - 3x + 2)$$

**Example # 20**

**Find the square root of $\frac{x^2}{4} + 4x^2 + \frac{ax^2}{3} + \frac{a^2}{9} - 2x^3 - \frac{4ax}{3}$**

**Solution:**

$$\frac{x^2}{4} + 4x^2 + \frac{ax^2}{3} + \frac{a^2}{9} - 2x^3 - \frac{4ax}{3}$$

The descending order of the expression are:

$$\frac{x^2}{4} - 2x^3 + 4x^2 + \frac{ax^2}{3} - \frac{4ax}{3} + \frac{a^2}{9}$$

Now

$$\begin{array}{r|l} & \quad \frac{x^2}{2} - 2x + \frac{a}{3} \\ \hline \frac{x^2}{2} & \frac{x^4}{4} - 2x^3 + 4x^2 + \frac{ax^2}{3} - \frac{4ax}{3} + \frac{a^2}{9} \\ & \pm \frac{x^4}{4} \\ \hline x^2 - 2x & -2x^3 + 4x^2 + \frac{ax^2}{3} - \frac{4ax}{3} + \frac{a^2}{9} \\ & \mp 2x^3 \pm 4x^2 \\ \hline x^2 - 4x + \frac{a}{3} & \frac{ax^2}{3} - \frac{4ax}{3} + \frac{a^2}{9} \\ & \pm \frac{ax^2}{3} \mp \frac{4ax}{3}x \pm \frac{a^2}{9} \\ \hline & 0 \end{array}$$

So

$$\sqrt{\frac{x^4}{4} - 2x^3 + 4x^2 + \frac{ax^2}{3} - \frac{4ax}{3} + \frac{a^2}{9}} = \pm\left(\frac{x^2}{2} - 2x + \frac{a}{3}\right)$$

**Ex # 6.3**

**Example # 24**

**What should be added to**

**What should be subtracted from**

**For what value of $x$**

**The expression $9x^4 - 12x^3 + 10x^2 - 3x - 3$**

**to make the perfect square**

**Solution:**

$$9x^4 - 12x^3 + 10x^2 - 3x - 3$$

$$\begin{array}{r|l} & \quad 3x^2 - 2x + 1 \\ \hline 3x^2 & 9x^4 - 12x^3 + 10x^2 - 3x - 3 \\ & \pm 9x^4 \\ \hline 6x^2 - 2x & -12x^3 + 10x^2 - 3x - 3 \\ & \mp 12x^3 \pm 4x^2 \\ \hline 6x^2 - 4x + 1 & 6x^2 - 3x - 3 \\ & \pm 6x^2 \mp 4x \pm 1 \\ \hline & x - 4 \end{array}$$

As for perfect square, Remainder $= 0$

$-x + 4$ should be Added to $9x^4 - 12x^3 + 10x^2 - 3x - 3$ will become perfect square.

$-x + 4 + (x - 4) = -x + 4 + x - 4$

$-x + 4 + (x - 4) = 0$

$x - 4$ should be Subtracted to $9x^4 - 12x^3 + 10x^2 - 3x - 3$ will become perfect square.

$x - 4 - (x - 4) = x - 4 - x + 4$

$x - 4 - (x - 4) = 0$

*For $x$*

$x - 4 = 0$

$x = 4$

# Chapter # 6



## Exercise# 6.3

**Page # 169**

**Q1: Find the square root by factorization method.**

$(i)\ \ \boldsymbol{x^2+4x+4}$

**Solution:**

$x^2+4x+4$

$x^2+4x+4=x^2+2(x)(2)+2^2$

$x^2+4x+4=(x+2)^2$

Taking Square on B.S

$\sqrt{x^2+4x+4}=\pm\sqrt{(x+2)^2}$

$\sqrt{x^2+4x+4}=\pm(x+2)$

---

$(ii)\ \ \boldsymbol{(x-y)^2+6(x-y)+9}$

**Solution:**

$(x-y)^2+6(x-y)+9$

$(x-y)^2+6(x-y)+9=(x-y)^2+2(x-y)(3)+3^2$

$(x-y)^2+6(x-y)+9=(x-y+3)^2$

Taking Square on B.S

$\sqrt{(x-y)^2+6(x-y)+9}=\pm\sqrt{(x-y+3)^2}$

$\sqrt{(x-y)^2+6(x-y)+9}=\pm(x-y+3)$

---

$(iii)\ \ \boldsymbol{x^2y^2-8xy+16}$

**Solution:**

$x^2y^2-8xy+16$

$x^2y^2-8xy+16=(xy)^2+2(xy)(4)+4^2$

$x^2y^2-8xy+16=(xy+4)^2$

Taking Square on B.S

$\sqrt{x^2y^2-8xy+16}=\pm\sqrt{(xy+4)^2}$

$\sqrt{x^2y^2-8xy+16}=\pm(xy+4)$

---

$(iv)\ \ \boldsymbol{x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+18}$

**Solution:**

$$x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+18$$

$$=x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+2+16$$

$$=x^2+\frac{1}{x^2}+2-8\left(x+\frac{1}{x}\right)+16$$

$$=\left(x+\frac{1}{x}\right)^2-2\left(x+\frac{1}{x}\right)(4)+(4)^2$$

$$=\left(x-\frac{1}{x}+4\right)^2$$

Now

$$x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+18=\left(x-\frac{1}{x}+4\right)^2$$

Taking Square on B.S

$$\sqrt{x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+18}=\pm\sqrt{\left(x-\frac{1}{x}+4\right)^2}$$

$$\sqrt{x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+18}=\pm\left(x-\frac{1}{x}+4\right)$$

---

$(v)\ \ \boldsymbol{(x+1)(x+2)(x+3)+1}$

**Solution:**

$x(x+1)(x+2)(x+3)+1$

Rearranging accordingly $0+3=1+2$

$=x(x+3)(x+1)(x+2)+1$

$=(x^2+3x)(x^2+2x+1x+2)+1$

$=(x^2+3x)(x^2+3x+2)+1$

$Let\ \ x^2+3x=y$

$=y^2+2y+1$

$=(y)^2+2(y)(1)+(1)^2$

$=(y+1)^2$

But $\mathrm{y}=x^2+3x$

$=(x^2+3x+1)^2$

Now

$x(x+1)(x+2)(x+3)+1=(x^2+3x+1)^2$

Taking Square on B.S

$\sqrt{x(x+1)(x+2)(x+3)+1}=\pm\sqrt{(x^2+3x+1)^2}$

$\sqrt{x(x+1)(x+2)(x+3)+1}=\pm(x^2+3x+1)$

# Chapter # 6

**Ex # 6.3**

$(vi)\quad \left(x+\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}$

**Solution:**

$$\left(x+\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}$$

$$=x^2+\frac{1}{x^2}+2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}$$

Subtract and Add 2

$$=x^2+\frac{1}{x^2}-2+2+2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}$$

$$=\left(x-\frac{1}{x}\right)^2+4-5\left(x-\frac{1}{x}\right)+\frac{9}{4}$$

$$=\left(x-\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}+4$$

$$=\left(x-\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9+16}{4}$$

$$=\left(x-\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{25}{4}$$

$$=\left(x-\frac{1}{x}\right)^2-2\left(x-\frac{1}{x}\right)\left(\frac{5}{2}\right)+\left(\frac{5}{2}\right)^2$$

$$=\left(x-\frac{1}{x}-\frac{5}{2}\right)^2$$

Now

$$\left(x+\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}=\left(x-\frac{1}{x}-\frac{5}{2}\right)^2$$

Taking square root on B.S

$$\sqrt{\left(x+\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}}=\pm\sqrt{\left(x-\frac{1}{x}-\frac{5}{2}\right)^2}$$

$$\sqrt{\left(x+\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}}=\pm\left(x-\frac{1}{x}-\frac{5}{2}\right)$$

---

$(vii)\quad \left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12$

**Solution:**

$$\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}+2\right)+12$$

**Ex # 6.3**

$$=\left(x^2+\frac{1}{x^2}\right)^2-4x^2-\frac{4}{x^2}-8+12$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}\right)-8+12$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}\right)+4$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-2\left(x^2+\frac{1}{x^2}\right)(2)+(4)^2$$

$$=\left(x^2+\frac{1}{x^2}-2\right)^2$$

Now

$$\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12=\left(x^2+\frac{1}{x^2}-2\right)^2$$

Taking square root on B.S

$$\sqrt{\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12}=\pm\sqrt{\left(x^2+\frac{1}{x^2}-2\right)^2}$$

$$\sqrt{\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12}=\pm\left(x^2+\frac{1}{x^2}-2\right)$$

---

$(viii)\quad \dfrac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^6}$

**Solution:**

$$\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^6}$$

$$=\frac{(2x^3)^2-2(2x^3)(3y^3)+(3y^3)^2}{(3x^2)^2+2(3x^2)(4y^2)+(4y^2)^2}$$

$$=\frac{(2x^3-3y^3)^2}{(3x^2+4y^2)^2}$$

$$=\left(\frac{2x^3-3y^3}{3x^2+4y^2}\right)^2$$

Now

$$\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^6}=\left(\frac{2x^3-3y^3}{3x^2+4y^2}\right)^2$$

Taking square root on B.S

$$\sqrt{\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^6}}=\pm\sqrt{\left(\frac{2x^3-3y^3}{3x^2+4y^2}\right)^2}$$

$$\sqrt{\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^6}}=\pm\left(\frac{2x^3-3y^3}{3x^2+4y^2}\right)$$

# Chapter # 6

## Ex # 6.3

**Q2: Find the square root of the following by Division method.**

$(i)$ $\boldsymbol{4x^4 - 4x^3 + 13x^2 - 6x + 9}$

**Solution:**

$$4x^4 - 4x^3 + 13x^2 - 6x + 9$$

$$\begin{array}{r|l}
 & 2x^2 - x + 3 \\ \hline
2x^2 & 4x^4 - 4x^3 + 13x^2 - 6x + 9 \\
 & \pm 4x^4 \\ \hline
4x^2 - x & -4x^3 + 13x^2 - 6x + 9 \\
 & \mp 4x^3 \pm x^2 \\ \hline
4x^2 - 2x + 3 & 12x^2 - 6x + 9 \\
 & \pm 12x^2 \mp 6x \pm 9 \\ \hline
 & 0
\end{array}$$

So

$$\sqrt{4x^4 - 4x^3 + 13x^2 - 6x + 9} = \pm(2x^2 - x + 3)$$

$(ii)$ $\boldsymbol{x^4 + x^3 - \frac{31}{4}x^2 - 4x + 16}$

**Solution:**

$$x^4 + x^3 - \frac{31}{4}x^2 - 4x + 16$$

$$\begin{array}{r|l}
 & x^2 + \frac{x}{2} - 4 \\ \hline
x^2 & x^4 + x^3 - \frac{31}{4}x^2 - 4x + 16 \\
 & \pm x^4 \\ \hline
2x^2 + \frac{x}{2} & x^3 - \frac{31}{4}x^2 - 4x + 16 \\
 & \pm x^3 \pm \frac{x^2}{4} \\ \hline
2x^2 + x - 4 & -8x^2 - 4x + 16 \\
 & \mp 8x^2 \mp 4x \pm 16 \\ \hline
 & 0
\end{array}$$

So

$$\sqrt{x^4 + x^3 - \frac{31}{4}x^2 - 4x + 16} = \pm\left(x^2 + \frac{x}{2} - 4\right)$$

$(iii)$ $\boldsymbol{x^2 - 2x + 1 + 2xy - 2y + y^2}$

**Solution:**

$$x^2 - 2x + 1 + 2xy - 2y + y^2$$

## Ex # 6.3

$$\begin{array}{r|l}
 & x - 1 + y \\ \hline
x & x^2 - 2x + 1 + 2xy - 2y + y^2 \\
 & \pm x^2 \\ \hline
2x - 1 & -2x + 1 + 2xy - 2y + y^2 \\
 & \mp 2x \pm 1 \\ \hline
2x - 2 + y & 2xy - 2y + y^2 \\
 & \pm 2xy \mp 2y \pm y^2 \\ \hline
 & 0
\end{array}$$

So

$$\sqrt{x^2 - 2x + 1 + 2xy - 2y + y^2} = \pm(x - 1 + y)$$

$(iv)$ $\boldsymbol{\left(x^2 - \frac{1}{x^2}\right)^2 - 12\left(x^2 - \frac{1}{x^2}\right) + 36}$

**Solution:**

$$\left(x^2 - \frac{1}{x^2}\right)^2 - 12\left(x^2 - \frac{1}{x^2}\right) + 36$$

$$= (x^2)^2 + \left(\frac{1}{x^2}\right)^2 - 2(x^2)\left(\frac{1}{x^2}\right) - 12x^2 + \frac{12}{x^2} + 36$$

$$= x^4 + \frac{1}{x^4} - 2 - 12x^2 + \frac{12}{x^2} + 36$$

Arrange it in ascending order

$$= x^4 - 12x^2 - 2 + 36 + \frac{12}{x^2} + \frac{1}{x^4}$$

$$= x^4 - 12x^2 + 34 + \frac{12}{x^2} + \frac{1}{x^4}$$

$$\begin{array}{r|l}
 & x^2 - 6 - \frac{1}{x^2} \\ \hline
x^2 & x^4 - 12x^2 + 34 + \frac{12}{x^2} + \frac{1}{x^4} \\
 & \pm x^4 \\ \hline
2x^2 - 6 & -12x^2 + 34 + \frac{12}{x^2} + \frac{1}{x^4} \\
 & \mp 12x^2 \pm 36 \\ \hline
2x^2 - 12 - \frac{1}{x^2} & -2 + \frac{12}{x^2} + \frac{1}{x^4} \\
 & \mp 2 \pm \frac{12}{x^2} \pm \frac{1}{x^4} \\ \hline
 & 0
\end{array}$$

So

$$\sqrt{\left(x^2 - \frac{1}{x^2}\right)^2 - 12\left(x^2 - \frac{1}{x^2}\right) + 36} = \pm\left(x^2 - 6 - \frac{1}{x^2}\right)$$

# Chapter # 6

**Ex # 6.3**

**Q3 (i): For what value of $k$ the expression**

$$\mathbf{4x^4 + 32x^2 + 96 + \frac{128}{x^2} + \frac{k}{x^4}}$$

**will become perfect square.**

**Solution:**

$4x^4 + 32x^2 + 96 + \frac{128}{x^2} + \frac{k}{x^4}$

$$\begin{array}{r|l}
 & 2x^2 + 8 + \frac{8}{x^2} \\
\hline
2x^2 & 4x^4 + 32x^2 + 96 + \frac{128}{x^2} + \frac{k}{x^4} \\
 & \pm 4x^4 \\
\hline
4x^2 + 8 & 32x^2 + 96 + \frac{128}{x^2} + \frac{k}{x^4} \\
 & \pm 32x^2 \pm 64 \\
\hline
4x^2 + 16 + \frac{8}{x^2} & 32 + \frac{128}{x^2} + \frac{k}{x^4} \\
 & \pm 32 \pm \frac{128}{x^2} \pm \frac{64}{x^4} \\
\hline
 & \frac{k}{x^4} - \frac{64}{x^4}
\end{array}$$

As for perfect square, Remainder $= 0$

$\frac{k}{x^4} - \frac{64}{x^4} = 0$

$\frac{k - 64}{x^4} = 0$

$k - 64 = 0 \times x^4$

$k - 64 = 0$

$k = 64$

**Q3 (ii):**

**(i) What should be added to**

**(ii) What should be subtracted to**

**(iii) For what value of $x$ the expression**

$\mathbf{4x^4 - 12x^3 + 17x^2 - 13x + 6}$ **so that it becomes perfect square**

**Solution:**

$4x^4 - 12x^3 + 17x^2 - 13x + 6$

$$\begin{array}{r|l}
 & 2x^2 - 3x + 2 \\
\hline
2x^2 & 4x^4 - 12x^3 + 17x^2 - 13x + 6 \\
 & \pm 4x^4 \\
\hline
4x^2 - 3x & -12x^3 + 17x^2 - 13x + 6 \\
 & \mp 12x^3 \pm 9x^2 \\
\hline
4x^2 - 6x + 2 & 8x^2 - 13x + 6 \\
 & \pm 8x^2 \mp 12x \pm 4 \\
\hline
 & -x + 2
\end{array}$$

As for perfect square, Remainder $= 0$

**Ex # 6.3**

$x - 2$ should be Added to $4x^4 - 12x^3 + 17x^2 - 13x + 6$ will become perfect square.

$-x + 2 + (x - 2) = -x + 2 + x - 2$

$-x + 2 + (x - 2) = 0$

$-x + 2$ should be Subtracted to $4x^4 - 12x^3 + 17x^2 - 13x + 6$ will become perfect square.

$-x + 2 - (-x + 2) = -x + 2 + x - 2$

$-x + 2 - (-x + 2) = 0$

*For x*

$-x + 2 = 0$

$-x = -2$

$x = 2$

**Q4: What should be subtracted and added to the expression $x^4 - 4x^3 + 10x + 7$ so that the expression is made perfect square?**

**Solution:**

$x^4 - 4x^3 + 10x + 7$

$$\begin{array}{r|l}
 & x^2 - 2x - 2 \\
\hline
x^2 & x^4 - 4x^3 + 10x + 7 \\
 & \pm x^4 \\
\hline
2x^2 - 2x & -4x^3 + 10x + 7 \\
 & \mp 4x^3 \quad \pm 4x^2 \\
\hline
2x^2 - 4x - 2 & -4x^2 + 10x + 7 \\
 & \mp 4x^2 \pm 8x \pm 4 \\
\hline
 & 2x + 3
\end{array}$$

As for perfect square, Remainder $= 0$

$-2x - 3$ should be Added to $x^4 - 4x^3 + 10x + 7$ will become perfect square.

$-2x - 3 + (2x + 3) = 2x + 3 - 2x - 3$

$-2x - 3 + (2x + 3) = 0$

$2x + 3$ should be Subtracted to $x^4 - 4x^3 + 10x + 7$ will become perfect square.

$2x + 3 - (2x + 3) = 2x + 3 - 2x - 3$

$2x + 3 - (2x + 3) = 0$

# Chapter # 6

**Ex # 6.3**

**Q5 (i): Find the value of $l$ and $m$ for which expression will become perfect square**

$\boldsymbol{x^4+4x^3+16x^2+lx+m}$

**Solution:**

$x^4+4x^3+16x^2+lx+m$

$$\begin{array}{r|l} & x^2+2x+6 \\ \hline x^2 & x^4+4x^3+16x^2+lx+m \\ & \pm x^4 \\ \hline 2x^2+2x & 4x^3+16x^2+lx+m \\ & \pm 4x^3 \pm 4x^2 \\ \hline 2x^2+4x+6 & 12x^2+lx+m \\ & \pm 12x^2 \pm 24x \pm 36 \\ \hline & lx-24x+m-36 \end{array}$$

As for perfect square, Remainder $=0$

$lx-24x+m-36=0$

$(l-24)x+(m-36)=0$

This $(l-24)x+(m-36)=0$ when

$(l-24)x+(m-36)=0x+0$

By compare the co-efficient of $x$ and constant

$$l-24=0$$
$$l=24$$

And $m-36=0$

$$m=36$$

Hence

$$l=24 \text{ and } m=36$$

**Q5 (ii): Find the value of $l$ and $m$ for which expression will become perfect square**

$\boldsymbol{49x^4-70x^3+109x^2+lx-m}$

**Solution:**

$49x^4-70x^3+109x^2+lx-m$

$$\begin{array}{r|l} & 7x^2-5x+6 \\ \hline 7x^2 & 49x^4-70x^3+109x^2+lx-m \\ & \pm 49x^4 \\ \hline 14x^2-5x & -70x^3+109x^2+lx-m \\ & \mp 70x^3 \pm 25x^2 \\ \hline 14x^2-10x+6 & 84x^2+lx-m \\ & \pm 84x^2 \mp 60x \pm 36 \\ \hline & lx+60x-m-36 \end{array}$$

As for perfect square, Remainder $=0$

$lx+60x-m-36=0$

**Ex # 6.3**

$(l+60)x+(-m-36)=0$

This $(l+60)x+(-m-36)=0$ when

$(l+60)x+(-m-36)=0x+0$

By compare the co-efficient of $x$ and constant

$$l+60=0$$
$$l=-60$$

And $-m-36=0$

$$-m=36$$
$$m=-36$$

Hence

$$l=-60 \text{ and } m=-36$$

# Review Exercise # 6

**Page # 171**

**Q2: Simplify the following.**

**(i):** $\boldsymbol{\dfrac{5}{2s+4}-\dfrac{3}{s^2+3s+2}+\dfrac{s}{s^2-s-2}}$

**Solution:**

$$\frac{5}{2s+4}-\frac{3}{s^2+3s+2}+\frac{s}{s^2-s-2}$$

$$=\frac{5}{2(s+2)}-\frac{3}{s^2+2s+1s+2}+\frac{s}{s^2-2s+1s-2}$$

$$=\frac{5}{2(s+2)}-\frac{3}{s(s+2)+1(s+2)}+\frac{s}{s(s-2)+1(s-2)}$$

$$=\frac{5}{2(s+2)}-\frac{3}{(s+2)(s+1)}+\frac{s}{(s-2)(s+1)}$$

$$=\frac{5(s+1)(s-2)-3\times2(s-2)+s\times2(s+2)}{2(s+2)(s+1)(s-2)}$$

$$=\frac{5(s^2-2s+1s-2)-6(s-2)+2s(s+2)}{2(s+2)(s+1)(s-2)}$$

$$=\frac{5(s^2-1s-2)-6s+12+2s^2+4s}{2(s+2)(s+1)(s-2)}$$

$$=\frac{5s^2-5s-10-6s+12+2s^2+4s}{2(s+2)(s+1)(s-2)}$$

$$=\frac{5s^2+2s^2-5s-6s+4s-10+12}{2(s+2)(s+1)(s-2)}$$

$$=\frac{7s^2-11s+4s-2}{2(s+2)(s+1)(s-2)}$$

$$=\frac{7s^2-7s-2}{2(s+2)(s+1)(s-2)}$$

# Chapter # 6

**Review Ex # 6**

$(ii).\ \dfrac{a}{(c-a)(a-b)}+\dfrac{b}{(a-b)(b-c)}+\dfrac{c}{(b-c)(c-a)}$

**Solution:**

$$\frac{a}{(c-a)(a-b)}+\frac{b}{(a-b)(b-c)}+\frac{c}{(b-c)(c-a)}$$

$$=\frac{a(b-c)+b(c-a)+c(a-b)}{(c-a)(a-b)(b-c)}$$

$$=\frac{ab-ac+bc-ab+ac-bc}{(a-b)(b-c)(c-a)}$$

$$=\frac{ab-ab-ac+ac+bc-bc}{(a-b)(b-c)(c-a)}$$

$$=\frac{0}{(a-b)(b-c)(c-a)}$$

$$=0$$

$(iii):\ \dfrac{x^2-4}{xy^2}\cdot\dfrac{2xy}{x^2-4x+4}$

**Solution:**

$$\frac{x^2-4}{xy^2}\cdot\frac{2xy}{x^2-4x+4}$$

$$=\frac{x^2-2^2}{xyy}\cdot\frac{2xy}{x^2-2(x)(2)+2^2}$$

$$=\frac{(x+2)(x-2)}{xyy}\cdot\frac{2xy}{(x+2)^2}$$

$$=\frac{(x+2)(x-2)}{xyy}\cdot\frac{2xy}{(x+2)(x+2)}$$

$$=\frac{(x-2)}{y}\cdot\frac{2}{(x+2)}$$

$$=\frac{2(x-2)}{y(x+2)}$$

$(iv):\ \dfrac{a^3-b^3}{a^4-b^4}\div\dfrac{a^2+ab+b^2}{a^2+b^2}$

**Solution:**

$$\frac{a^3-b^3}{a^4-b^4}\div\frac{a^2+ab+b^2}{a^2+b^2}$$

$$=\frac{a^3-b^3}{a^4-b^4}\times\frac{a^2+b^2}{a^2+ab+b^2}$$

**Review Ex # 6**

$$=\frac{(a-b)(a^2+ab+b^2)}{(a^2+b^2)(a^2-b^2)}\times\frac{a^2+b^2}{a^2+ab+b^2}$$

$$=\frac{(a-b)(a^2+ab+b^2)}{(a^2+b^2)(a+b)(a-b)}\times\frac{a^2+b^2}{a^2+ab+b^2}$$

$$=\frac{1}{a+b}\times\frac{1}{1}$$

$$=\frac{1}{a+b}$$

# Chapter # 6

**Review Ex # 6**

**Q3:** **Find L.C.M of $x^3-6x^2+11x-6$ and $x^3-4x+3$**

$x^3-6x^2+11x-6$ and $x^3-4x+3$

**Solution:**

$Let\ A = x^3-6x^2+11x-6$

$and\ B = x^3-4x+3$

As we have:

$$L.C.M = \frac{A\times B}{H.C.F} --equ\ (i)$$

First we find H.C.F

$$\begin{array}{l}
x^3-4x+3 \,\big)\, x^3-6x^2+11x-6 \,\big(\, 1 \\
\qquad\qquad\quad \pm x^3 \qquad \mp 4x \pm 3 \\
\hline
-3 \,\big)\, -6x^2+15x-9 \\
\qquad 2x^2-5x+3 \,\big)\, x^3-4x+3 \,\big(\, x+5 \\
\qquad\qquad\qquad \times 2 \\
\hline
\qquad\qquad\qquad 2x^3-8x+6 \\
\qquad\qquad\qquad \pm 2x^3 \pm 3x \quad \mp 5x^2 \\
\hline
\qquad\qquad\qquad 5x^2-11x+6 \\
\qquad\qquad\qquad \times 2 \\
\hline
\qquad\qquad\qquad 10x^2-22x+12 \\
\qquad\qquad\qquad \pm 10x^2 \mp 25x \pm 15 \\
\hline
\qquad\qquad\qquad 3 \,\big)\, 3x-3 \\
\hline
\qquad\qquad\qquad\qquad x-1 \,\big)\, 2x^2-5x+3 \,\big(\, 2x-3 \\
\qquad\qquad\qquad\qquad\qquad \pm 2x^2 \mp 2x \\
\hline
\qquad\qquad\qquad\qquad\qquad -3x+3 \\
\qquad\qquad\qquad\qquad\qquad \mp 3x \pm 3 \\
\hline
\qquad\qquad\qquad\qquad\qquad \times
\end{array}$$

$H.C.F = x-1$

Now put the values in equ (i)

$$L.C.M = \frac{(x^3-6x^2+11x-6\,)(x^3-4x+3)}{x-1}$$

Now by Simple Division

$$\begin{array}{l}
\qquad\qquad x^2-5x+6 \\
x-1 \,\big)\, x^3-6x^2+11x-6 \\
\qquad \pm x^3 \mp x^2 \\
\hline
\qquad\qquad -5x^2+11x-6 \\
\qquad\qquad \mp 5x^2 \pm 5x \\
\hline
\qquad\qquad\qquad 6x-6 \\
\qquad\qquad\qquad \pm 6x \mp 6 \\
\hline
\qquad\qquad\qquad \times
\end{array}$$

$So\ L.C.M = (x^2-5x+6)(x^3-4x+3)$

# Chapter # 6

## Review Ex # 6

**Q4: Find the square root of :**

(i): $\boldsymbol{4x^2 - 12x + 9}$

**Solution:**

$4x^2 - 12x + 9$

$4x^2 - 12x + 9 = (2x)^2 - 2(2x)(3) + (3)^2$

$4x^2 - 12x + 9 = (2x - 3)^2$

Taking Square on B.S

$\sqrt{4x^2 - 12x + 9} = \pm\sqrt{(2x - 3)^2}$

$\sqrt{4x^2 - 12x + 9} = \pm(2x - 3)$

## Review Ex # 6

(ii): $\boldsymbol{x^4 + 4x^3 + 6x^2 + 4x + 1}$

**Solution:**

$x^4 + 4x^3 + 6x^2 + 4x + 1$

$$
\begin{array}{r|l}
 & x^2 + 2x + 1 \\
\hline
x^2 & x^4 + 4x^3 + 6x^2 + 4x + 1 \\
 & \pm x^4 \\
\hline
2x^2 + 2x & 4x^3 + 6x^2 + 4x + 1 \\
 & \pm 4x^3 \pm 4x^2 \\
\hline
2x^2 + 4x + 1 & 2x^2 + 4x + 1 \\
 & \pm 2x^2 \pm 4x \pm 1 \\
\hline
 & 0
\end{array}
$$

So

$$\sqrt{x^4 + 4x^3 + 6x^2 + 4x + 1} = \pm(\,x^2 + 2x + 1)$$

**Think**

$Q5$: **Simplify** $\boldsymbol{\dfrac{x^3 - y^3}{x^3 - z^3} \times \dfrac{x^2 + xy + xz + yz}{x^4 + x^2y^2 + y^4} \times \dfrac{x^3 + y^3}{x^2 - y^2}}$

**Solution:**

$$\frac{x^3 - y^3}{x^3 + z^3} \times \frac{x^2 + xy + xz + yz}{x^4 + x^2y^2 + y^4} \times \frac{x^3 + y^3}{x^2 - y^2}$$

$$= \frac{(x - y)(x^2 + xy + y^2)}{(x + z)(x^2 - xz + z^2)} \times \frac{x(x + y) + z(x + y)}{x^4 + y^4 + x^2y^2} \times \frac{(x + y)(x^2 - xy + y^2)}{(x + y)(x - y)}$$

$$= \frac{(x^2 + xy + y^2)}{(x^2 - xz + z^2)} \times \frac{(x + y)}{(x^2)^2 + (y^2)^2 + 2x^2y^2 - 2x^2y^2 + x^2y^2} \times \frac{(x^2 - xy + y^2)}{1}$$

$$= \frac{(x^2 + xy + y^2)}{(x^2 - xz + z^2)} \times \frac{(x + y)(x^2 - xy + y^2)}{(x^2 + y^2)^2 - x^2y^2}$$

$$= \frac{(x^2 + xy + y^2)}{(x^2 - xz + z^2)} \times \frac{(x + y)(x^2 - xy + y^2)}{(x^2 + y^2)^2 - (xy)^2}$$

$$= \frac{(x^2 + xy + y^2)}{(x^2 - xz + z^2)} \times \frac{(x + y)(x^2 - xy + y^2)}{(x^2 + y^2 + xy)(x^2 + y^2 - xy)}$$

$$= \frac{1}{(x^2 - xz + z^2)} \times \frac{(x + y)}{1}$$

$$= \frac{(x + y)}{(x - z)(x^2 + xz + z^2)}$$

Tehkals.com
Learn & Teach

# MATHEMATICS

## Class 9th (KPK)

## Chapter # 7 Linear Equations & Inequealites

NAME: ______________________________

F.NAME: ______________________________

CLASS:_____________ SECTION: _________

ROLL #: ______ SUBJECT: ______________

ADDRESS: _________________________________________

_________________________________________________

SCHOOL: ___________________________________________

# Chapter # 7

# UNIT # 7

## LINEAR EQUATIONS AND INEQULITIES

### Ex # 7.1

**Linear equation**

An equation the highest degree or exponent of a variable is one is called linear equation.

**Linear equation in one variable**

A linear equation in which one variable is used is called linear equation in one variable.

**General form**

$$ax + b = 0$$

**Example:**

$2x + 3 = 0$

$\frac{5}{2}y - 4 = 0$

$5x - 15 = 2x + 3$

**Solution of Linear Equation**

To solve the linear equation, follow the following steps.

First solve the brackets if any

Now shift the constant term to other side of equation by adding or subtracting to B.S

Transfer all terms containing variable on one side and simplify them if any.

Divide or multiply both sides of the equation by the co – efficient of the variable.

At last, sing numerical value is obtained.

Verify by putting the value in original equation.

پہلے Brackets کو Solve کریں۔

پھر constant term کو دوسرے طرف Shift کریں Add یا Subtract کر کے

Variable والے Term کو بھی ایک طرف Shift کریں

Equation کے دونوں طرف Variable کے Co – efficient کے

ساتھ Multiply یا Divide کریں

**Example # 2**

**Solve $2x + 3 = 1 - (x - 1)$**

**Solution:**

$2x + 3 = 1 - 6(x - 1) \ldots\ldots equ(i)$

$2x + 3 = 1 - 6x + 6$

$2x + 3 = -6x + 1 + 6$

### Ex # 7.1

$2x + 3 = -6x + 7$

Subtract 3 from B.S

$2x + 3 - 3 = -6x + 7 - 3$

$2x = -6x + 4$

*Add 6x on B.S*

$2x + 6x = -6x + 6x + 4$

$8x = 4$

Divide B.S by 8

$\frac{8x}{8} = \frac{4}{8}$

$x = \frac{1}{2}$

**Verification**

Put $x = \frac{1}{2}$ in equ (i)

$2\left(\frac{1}{2}\right) + 3 = 1 - 6\left(\frac{1}{2} - 1\right)$

$1 + 3 = 1 - 6\left(\frac{1 - 2}{2}\right)$

$4 = 1 - 6\left(\frac{-1}{2}\right)$

$4 = 1 - 3(-1)$

$4 = 1 + 3$

$4 = 4$

Thus Solution Set $= \left\{\frac{1}{2}\right\}$

**Example # 3**

***Solve*** $\boldsymbol{3x + \frac{x}{5} - 5 = \frac{1}{5} + 5x}$

**Solution:**

$3x + \frac{x}{5} - 5 = \frac{1}{5} + 5x \ldots\ldots equ(i)$

Separate the variable and constant

$3x + \frac{x}{5} - 5x = \frac{1}{5} + 5$

$3x - 5x + \frac{x}{5} = \frac{1}{5} + 5$

$\frac{x}{5} + 3x - 5x = \frac{1}{5} + 5$

$\frac{x}{5} - 2x = \frac{1}{5} + 5$

**Ex # 7.1**

$$\frac{x-10x}{5}=\frac{1+25}{5}$$
$$\frac{-9x}{5}=\frac{26}{5}$$
Multiply B.S by 5
$$5\times\frac{-9x}{5}=5\times\frac{26}{5}$$
$$-9x=26$$
Divide B.S by -9
$$\frac{-9x}{-9}=\frac{26}{-9}$$
$$x=-\frac{26}{9}$$
**Verification**

Put $x=-\frac{26}{9}$ in equ (i)
$$3\left(-\frac{26}{9}\right)+\frac{-\frac{26}{9}}{5}-5=\frac{1}{5}+5\left(-\frac{26}{9}\right)$$
$$-\frac{26}{3}+\left(-\frac{26}{9}\right)\div 5-5=\frac{1}{5}-\frac{130}{9}$$
$$-\frac{26}{3}-\frac{26}{9}\times\frac{1}{5}-5=\frac{1}{5}-\frac{130}{9}$$
$$-\frac{26}{3}-\frac{26}{45}-5=\frac{1}{5}-\frac{130}{9}$$
$$\frac{-390-26-225}{45}=\frac{9-650}{45}$$
$$\frac{-641}{45}=\frac{-641}{45}$$
Thus Solution Set $=\left\{-\frac{26}{9}\right\}$

**Example # 4**

**Age of mother is 13 time the age of her daughter. It will be only five times after four years. Find their present ages.**

**Solution**:

Let the present age of daughter $= x$ years
So the present age of mother $= 13x$ years
After four years
Age of daughter $=(x+4)years$
and age of mother $=(13x+4)years$
According to condition
Age of mother $= 5$(Age of daughter)
$$13x+4=5(x+4)$$
$$13x+4=5x+20$$

**Ex # 7.1**

Now shift the variable and constant
$$13x-5x=20-4$$
$$8x=16$$
Divide B.S by 8
$$\frac{8x}{8}=\frac{16}{8}$$
$$x=2$$
Thus present age of daughter $= x = 2years$
And present age of mother $= 13\times 2$
$= 26years$

**Example # 5**

**A number consist of two digits. The sum of digits is 8. If digits are interchanged, then new number becomes 36 less than the original numbers. Find the number.**

**Solution**:

$Let\ digit\ at\ ones/unit\ place = x$
$And\ digit\ at\ tens\ place = y$
$So\ the\ original\ number\ = 10\times y+1\times x$
$= 10y+x$
$If\ place\ of\ digits\ are\ interchanged$
$New\ number = 10\times x+1\times y$
$= 10x+y$
According to given conditions
Sum of digits is 8
So,
$x+y=8\ \ldots\ldots equ(i)$
And
$New\ number = Original\ number - 36$
$$10x+y=10y+x-36$$
$$10x-x=10y-y-36$$
$$9x=9y-36$$
$$9x=9(y-4)$$
Divide B.S by 9
$$\frac{9x}{9}=\frac{9(y-4)}{9}$$
$x=y-4\ \ldots\ldots equ(ii)$
Put $x=y-4$in equ (i)
$$y-4+y=8$$
Add 4 on B.S
$$y-4+4+y=8+4$$
$$y+y=12$$
$$2y=12$$

**Ex # 7.1**

Divide B.S by 2

$$\frac{2y}{2}=\frac{12}{2}$$

$y=6$

Put $y=6$in equ (ii)

$x=6-4$

$x=2$

$$\begin{aligned} As\ the\ Original\ number &= 10y+x \\ &= 10(6)+2 \\ &= 60+2 \\ &= 62 \end{aligned}$$

# Exercise # 7.1

**Page # 177**

**Q1: Find the solution sets of the following equations and verify the answer.**

**(i)** $\mathbf{5x+8=23}$

**Solution:**

$5x+8=23 \ldots\ldots equ(i)$

Subtract 8 from B.S

$5x+8-8=23-8$

$5x=15$

Divide 5 on B.S

$$\frac{5x}{5}=\frac{15}{5}$$

$x=3$

**Verification**

Put $x=3$ in equ (i)

$5(3)+8=23$

$15+8=23$

$23=23$

Thus Solution Set $=\{3\}$

**(ii)** $\mathbf{\frac{3}{5}x-\frac{2}{3}=2}$

**Solution:**

$\frac{3}{5}x-\frac{2}{3}=2 \ldots\ldots equ(i)$

$$\frac{9x-10}{15}=2$$

Multiply 15 on B.S

$$\frac{9x-10}{15}\times 15=2\times 15$$

$9x-10=30$

**Ex # 7.1**

Add 10 on B.S

$9x-10+10=30+10$

$9x=40$

Divide 9 on B.S

$$\frac{9x}{9}=\frac{40}{9}$$

$$x=\frac{40}{9}$$

**Verification**

Put $x=\frac{40}{9}$ in equ (i)

$$\frac{3}{5}\times\frac{40}{9}-\frac{2}{3}=2$$

$$\frac{1}{1}\times\frac{8}{3}-\frac{2}{3}=2$$

$$\frac{8}{3}-\frac{2}{3}=2$$

$$\frac{8-2}{3}=2$$

$$\frac{6}{3}=2$$

$2=2$

Thus Solution Set $=\left\{\frac{40}{9}\right\}$

**(iii)** $\mathbf{6x-5=2x+9}$

**Solution:**

$6x-5=2x+9 \ldots\ldots equ(i)$

Add 5 on B.S

$6x-5+5=2x+9+5$

$6x=2x+14$

Subtract $2x$ from B.S

$6x-2x=2x-2x+14$

$4x=14$

Divide B.S by 4

$$\frac{4x}{4}=\frac{14}{4}$$

$$x=\frac{7}{2}$$

**Verification**

Put $x=\frac{7}{2}$ in equ (i)

$$6\left(\frac{7}{2}\right)-5=2\left(\frac{7}{2}\right)+9$$

$3(7)-5=7+9$

$21-5=16$

$16=16$

# Chapter # 7

## Ex # 7.1

Thus Solution Set $= \left\{\frac{7}{2}\right\}$

**(iv)** $\boldsymbol{\frac{2}{x-1} = \frac{1}{x-2}}$

**Solution:**

$\frac{2}{x-1} = \frac{1}{x-2} \ldots\ldots equ(i)$

By Cross Multiplication

$2(x-2) = 1(x-1)$

$2x - 4 = x - 1$

Add 4 on B.S

$2x - 4 + 4 = x - 1 + 4$

$2x = x + 3$

Subtract $x$ from B.S

$2x - x = x - x + 3$

$x = 3$

**Verification**

Put $x = 3$ in equ (i)

$\frac{2}{3-1} = \frac{1}{3-2}$

$\frac{2}{2} = \frac{1}{1}$

$1 = 1$

Solution Set $= \{3\}$

**(v)** $\boldsymbol{\frac{1}{7x+13} = \frac{2}{9}}$

**Solution:**

$\frac{1}{7x+13} = \frac{2}{9} \ldots\ldots equ(i)$

By Cross Multiplication

$1 \times 9 = 2(7x + 13)$

$9 = 14x + 26$

Subtract 26 from B.S

$9 - 26 = 14x - 26$

$-17 = 14x$

Divide B.S by 14

$\frac{-17}{14} = \frac{14x}{14}$

$\frac{-17}{14} = x$

$x = \frac{-17}{14}$

**Verification**

## Ex # 7.1

Put $x = \frac{-17}{14}$ in equ (i)

$\frac{1}{7\left(\frac{-17}{14}\right) + 13} = \frac{2}{9}$

$\frac{1}{\frac{-17}{2} + 13} = \frac{2}{9}$

$\frac{1}{\frac{-17+26}{2}} = \frac{2}{9}$

$\frac{1}{\frac{9}{2}} = \frac{2}{9}$

$1 \div \frac{9}{2} = \frac{2}{9}$

$1 \times \frac{2}{9} = \frac{2}{9}$

$\frac{2}{9} = \frac{2}{9}$

Solution Set $= \left\{\frac{-17}{14}\right\}$

**(vi)** $\boldsymbol{10(x-4) = 4(2x-1) + 5}$

**Solution:**

$10(x-4) = 4(2x-1) + 5 \ldots\ldots equ(i)$

$10x - 40 = 8x - 4 + 5$

$10x - 40 = 8x + 1$

$10x - 40 = 8x + 1$

Add 40 on B.S

$10x - 40 + 40 = 8x + 1 + 40$

$10x = 8x + 41$

Subtract $8x$ from B.S

$10x - 8x = 8x - 8x + 41$

$2x = 41$

Divide B.S by 2

$\frac{2x}{2} = \frac{41}{2}$

$x = \left\{\frac{41}{2}\right\}$

**Verification**

Put $x = \frac{41}{2}$ in equ (i)

$10\left(\frac{41}{2} - 4\right) = 4\left(2 \times \frac{41}{2} - 1\right) + 5$

$10\left(\frac{41-8}{2}\right) = 4(41-1) + 5$

$10\left(\frac{33}{2}\right) = 4(40) + 5$

# Chapter # 7

**Ex # 7.1**

$5(33) = 160 + 5$
$165 = 165$
$\text{Solution Set} = \frac{41}{2}$

**Q2: Awais thought of a number, add 3 with it. Then he doubled the sum. He got 40. What was the original number?**

**Solution:**

Let the number $= x$
As the given condition is defined as
Add 3 and double the sum got 40
So, we get
$2(x+3) = 40$
Divide B.S by 2
$\frac{2(x+3)}{2} = \frac{40}{2}$
$x + 3 = 20$
Subtract 3 from B.S
$x + 3 - 3 = 20 - 3$
$x = 17$
Thus, the original number $= 17$

**Q3: The sum of two numbers is -4 and their difference is 6. What are the numbers?**

**Solution:**

$Let\ the\ two\ numbers\ are\ x\ and\ y$
According to first condition
The sum of two numbers is $-4$
So,
$x + y = -4 \ldots\ldots equ(i)$
According to second condition
The difference of two numbers is 6
So,
$x - y = 6 \ldots\ldots equ(ii)$
Now add equ(i) and equ (ii)
$x + y + x - y = -4 + 6$
$x + x + y - y = 2$
$2x = 2$
Divide B.S by 2
$\frac{2x}{2} = \frac{2}{2}$
$x = 1$
Put $x = 1$ in equ (i)
$1 + y = -4$

**Ex # 7.1**

Subtract 1 from B.S
$1 - 1 + y = -4 - 1$
$y = -5$
$Thus\ the\ two\ numbers\ are\ 1\ and\ -5$

**Q4: The sum of three consecutive odd integers is 81. Find the numbers.**

**Solution:**

As the difference is 2 between two consecutive odd integers
$Let\ first\ odd\ integer = x$
$Second\ odd\ integer = x + 2$
$And\ third\ odd\ integer = x + 4$
According to given condition
The sum of three consecutive odd integers is 81
So,
$x + x + 2 + x + 4 = 81$
$x + x + x + 2 + 4 = 81$
$3x + 6 = 81$
Subtract 6 from B.S
$3x + 6 - 6 = 81 - 6$
$3x = 75$
Divide B.S by 3
$\frac{3x}{3} = \frac{75}{3}$
$x = 25$
$Let\ first\ odd\ integer = x = 25$
$Second\ odd\ integer = x + 2$
$= 25 + 2$
$= 27$
$And\ third\ odd\ integer = x + 4$
$= 25 + 4$
$= 29$
So the consecutive odd integers are 25, 27 and 29

# Chapter # 7

## Ex # 7.1

**Q5:** **A man is 41 year old and his son is 9 year old. In how many years will the father be three times as old as the son?**

**Solution:**

$let\ father's\ age = 41\ years$

$and\ son'sage = 9\ years$

$Let\ the\ required\ years = x$

$So\ after\ x\ years$

$Father's\ age = 41 + x$

$Son's\ age = 9 + x$

According to given condition

$Age\ of\ father = 3(Age\ of\ son)$

$41 + x = 3(9 + x)$

$41 + x = 27 + 3x$

$41 - 27 = 3x - x$

$14 = 2x$

$2x = 14$

Divide B.S by 2

$\frac{2x}{2} = \frac{14}{2}$

$x = 7$

So the required number of years=7

Thus after 7 years father's age will be three times as his son

**Q6:** **The tens digit of a certain two – digitnumber exceeds the unit digit by 4 and is 1 less than twice the ones digit. Find the number.**

**Solution:**

$Let\ digit\ at\ ones/unit\ place = x$

$And\ digit\ at\ tens\ place = y$

$So\ two\ digit\ number\ = 10 \times y + 1 \times x$

$= 10y + x$

According to given conditions

Tens digit exceeds the unit digit by 1

So,

$Tens\ digit = Ones\ digit + 4$

$y = x + 4 \dots \dots equ(i)$

And

Tens digit is 1 less than twice the ones digits

So,

$Tens\ digit = twice\ the\ one\ digit - 1$

$y = 2x - 1 \dots \dots equ(ii)$

## Ex # 7.1

Compare equ (i) and (ii), we get

$x + 4 = 2x - 1$

$4 + 1 = 2x - x$

$5 = x$

$x = 5$

Put $x = 5$in equ (i)

$y = 5 + 4$

$y = 9$

$Thus\ the\ two\ digit = 10y + x$

$= 10(9) + 5$

$= 90 + 5$

$= 95$

**Q7:** **The sum of two digits is 10. It the place of digits are changed then the new number is decreased by 18. Find the numbers.**

**Solution:**

$Let\ digit\ at\ ones/unit\ place = x$

$And\ digit\ at\ tens\ place = y$

$So\ the\ original\ number\ = 10 \times y + 1 \times x$

$= 10y + x$

If place of digits are interchanged

New number $= 10 \times x + 1 \times y$

$= 10x + y$

According to given conditions

Sum of digits is 10

So,

$x + y = 10 \dots \dots equ(i)$

And

$New\ number = Original\ number - 18$

$10x + y = 10y + x - 18$

$10x - x = 10y - y - 18$

$9x = 9y - 18$

$9x = 9(y - 2)$

Divide B.S by 9

$\frac{9x}{9} = \frac{9(y-2)}{9}$

$x = y - 2 \dots \dots equ(ii)$

Put $x = y - 2$in equ (i)

$y - 2 + y = 10$

Add 2 on B.S

$y - 2 + 2 + y = 10 + 2$

$y + y = 12$

$2y = 12$

# Chapter # 7

## Ex # 7.1

Divide B.S by 2

$$\frac{2y}{2}=\frac{12}{2}$$

$y=6$

Put $y=6$ in equ (ii)

$x=6-2$

$x=4$

$$As\ the\ Original\ number=10y+x$$
$$=10(6)+4$$
$$=60+4$$
$$=64$$

**Q8: It the breadth of the room is one fourth of its length and the perimeter of the room is 20m. Find length and breadth of the room.**

**Solution:**

$Let\ length\ of\ room=x\ m$

As breadth is one fourth of its length

$Then\ breadth\ of\ room=\frac{x}{4}\ m$

As Perimeter of room=20 m

As we know that

$$P=2(l+2)$$

Put the values

$$20=2\left(x+\frac{x}{4}\right)$$

$$20=2\left(\frac{4x+x}{4}\right)$$

$$20=\frac{5x}{2}$$

$Multiply\ B.S\ by\ \frac{2}{5}$

$$20\times\frac{2}{5}=\frac{5x}{2}\times\frac{2}{5}$$

$4\times 2=x$

$8=x$

$x=8$

Thus

$Let\ length\ of\ room=x\ m=8m$

$$breadth\ of\ room=\frac{x}{4}\ m$$
$$=\frac{8}{4}\ m$$
$$=2\ m$$



## Ex # 7.2

**Radical equation**

An equation in which the variable occurs under a radical is called radical equation.

**Note:**

The radicand should be a variable (unknown).

$\sqrt{x}+5=9$ is a radical equation but $2x+\sqrt{5}=9$ is not a radical equation.

The radical equation will be considered as positive numbers.

$\sqrt{x+6}=-11$ has no real solution and is not true for any value of $x$.

**Example # 5**

**$Solve\ \sqrt{2x}+5=9$**

**Solution:**

$\sqrt{2x}+5=9\ ……equ(i)$

Subtract 5 from B.S

$\sqrt{2x}+5-5=9-5$

$\sqrt{2x}=4$

Taking square on B.S

$\left(\sqrt{2x}\right)^2=(4)^2$

$2x=16$

Divide B.S by 2

$$\frac{2x}{2}=\frac{16}{2}$$

$x=8$

**Verification**

Put $x=8$ in equ (i)

$\sqrt{2(8)}+5=9$

$\sqrt{16}+5=9$

$4+5=9$

$9=9$

Thus Solution Set $=\{8\}$

**Example # 7**

**$\sqrt{3x-2}=\sqrt{5x+4}$**

**Solution:**

$\sqrt{3x-2}=\sqrt{5x+4}$

$\sqrt{3x-2}=\sqrt{5x+4}\ ……equ(i)$

Take square root on B.S

$\left(\sqrt{3x-2}\right)^2=\left(\sqrt{5x+4}\right)^2$

$3x-2=5x+4$

Subtract $5x$ from B.S

$3x - 5x - 2 = 5x - 5x + 4$

$-2x - 2 = 4$

Add 2 on B.S

$-2x - 2 + 2 = 4 + 2$

$-2x = 6$

Divide B.S by $-2$

$\frac{-2x}{-2} = \frac{6}{-2}$

$x = -3$

**Verification**

Put $x = -3$ in equ (i)

$\sqrt{3(-3) - 2} = \sqrt{5(-3) + 4}$

$\sqrt{-9 - 2} = \sqrt{-15 + 4}$

$\sqrt{-11} = \sqrt{-11}$

Thus Solution Set $= \{-3\}$

**Example # 8**

$\boldsymbol{\sqrt{3x + 2} + 6 = 2}$

**Solution:**

$\sqrt{3x + 2} + 6 = 2 \ldots \ldots equ(i)$

Subtract 6 from B.S

$\sqrt{3x + 2} + 6 - 6 = 2 - 6$

$\sqrt{3x + 2} = -4$

Taking square on B.S

$\left(\sqrt{3x + 2}\right)^2 = (-4)^2$

$3x + 2 = 16$

Subtract 2 from B.S

$3x + 2 - 2 = 16 - 2$

$3x = 14$

Divide B.S by 3

$\frac{3x}{3} = \frac{14}{3}$

$x = \frac{14}{3}$

Solution Set $= \{\ \}$

**Verification**

Put $x = \frac{14}{3}$ in equ (i)

$\sqrt{3\left(\frac{14}{3}\right) + 2} + 6 = 2$

$\sqrt{14 + 2} + 6 = 2$



$\sqrt{16} + 6 = 2$

$4 + 6 = 2$

$10 = 2$

Hence

$10 \neq 2$

Thus the given equation has no solution.

Solution Set $= \{\ \}$

# Exercise # 7.2

**Page # 180**

**Q: Solve the following radical equation.**

**1.** $\boldsymbol{2\sqrt{a} - 3 = 7}$

**Solution:**

$2\sqrt{a} - 3 = 7 \ldots \ldots equ(i)$

Add 3 on B.S

$2\sqrt{a} - 3 + 3 = 7 + 3$

$2\sqrt{a} = 10$

Divide B.S by 2

$\frac{2\sqrt{a}}{2} = \frac{10}{2}$

$\sqrt{a} = 5$

Taking square on B.S

$\left(\sqrt{a}\right)^2 = (5)^2$

$a = 25$

**Verification**

Put $a = 25$ in equ (i)

$2\sqrt{25} - 3 = 7$

$2(5) - 3 = 7$

$10 - 3 = 7$

$7 = 7$

Thus Solution Set $= \{25\}$

**2.** $\boldsymbol{8 + 3\sqrt{b} = 20}$

**Solution:**

$8 + 3\sqrt{b} = 20 \ldots \ldots equ(i)$

Subtract 8 from B.S

$8 - 8 + 3\sqrt{b} = 20 - 8$

$3\sqrt{b} = 12$

Divide B.S by 3

$\frac{3\sqrt{b}}{3} = \frac{12}{3}$

$\sqrt{b} = 4$

**Ex # 7.2**

Taking square on B.S

$\left(\sqrt{b}\right)^2 = (4)^2$

$b = 16$

**Verification**

Put $b = 16$ in equ (i)

$8 + 3\sqrt{16} = 20$

$8 + 3(4) = 20$

$8 + 12 = 20$

$20 = 20$

Thus Solution Set $= \{16\}$

**3.** $\mathbf{7 - \sqrt{2b} = 3}$

**Solution:**

$7 - \sqrt{2b} = 3 \ldots \ldots equ(i)$

Subtract 7 from B.S

$7 - 7 - \sqrt{2b} = 3 - 7$

$-\sqrt{2b} = -4$

$\sqrt{2b} = 4$

Taking square on B.S

$\left(\sqrt{2b}\right)^2 = (4)^2$

$2b = 16$

Divide B.S by 2

$\frac{2b}{2} = \frac{16}{2}$

$b = 8$

**Verification**

Put $b = 8$ in equ (i)

$7 - \sqrt{2(8)} = 3$

$7 - \sqrt{16} = 3$

$7 - 4 = 3$

$3 = 3$

Thus Solution Set $= \{8\}$

**4.** $\mathbf{\sqrt{r} - 5 = \sqrt{r} + 9}$

**Solution:**

$8\sqrt{r} - 5 = \sqrt{r} + 9 \ldots \ldots equ(i)$

Add 5 on B.S

$8\sqrt{r} - 5 + 5 = \sqrt{r} + 9 + 5$

$8\sqrt{r} = \sqrt{r} + 14$

Subtract $\sqrt{r}$ from B.S

$8\sqrt{r} - \sqrt{r} = \sqrt{r} - \sqrt{r} + 14$

$7\sqrt{r} = 14$

**Ex # 7.2**

Divide B.S by 7

$\frac{7\sqrt{r}}{7} = \frac{14}{7}$

$\sqrt{r} = 2$

Taking square on B.S

$\left(\sqrt{r}\right)^2 = (2)^2$

$r = 4$

**Verification**

Put $r = 4$ in equ (i)

$7\sqrt{4} - 5 = \sqrt{4} + 9$

$7(2) - 5 = 2 + 9$

$14 - 5 = 11$

$11 = 11$

Thus Solution Set $= \{4\}$

**5.** $\mathbf{20 - 3\sqrt{t} = \sqrt{t} - 4}$

**Solution:**

$20 - 3\sqrt{t} = \sqrt{t} - 4 \ldots \ldots equ(i)$

Subtract 20 from B.S

$20 - 20 - 3\sqrt{t} = \sqrt{t} - 4 - 20$

$-3\sqrt{t} = \sqrt{t} - 24$

Subtract $\sqrt{t}$ from B.S

$-3\sqrt{t} - \sqrt{t} = \sqrt{t} - \sqrt{t} - 24$

$-4\sqrt{t} = -24$

$4\sqrt{t} = 24$

Divide B.S by 4

$\frac{4\sqrt{t}}{4} = \frac{24}{4}$

$\sqrt{t} = 6$

Taking square on B.S

$\left(\sqrt{t}\right)^2 = (6)^2$

$t = 36$

**Verification**

Put $t = 36$ in equ (i)

$20 - 3\sqrt{36} = \sqrt{36} - 4$

$20 - 3(6) = 6 - 4$

$20 - 18 = 2$

$2 = 2$

Thus Solution Set $= \{36\}$

# Chapter # 7

**Ex # 7.2**

6. $\mathbf{2\sqrt{5x} - 3 = 7}$

**Solution:**

$2\sqrt{5x} - 3 = 7 \dots\dots equ(i)$

Add 3 on B.S

$2\sqrt{5x} - 3 + 3 = 7 + 3$

$2\sqrt{5x} = 10$

Divide B.S by 2

$\frac{2\sqrt{5x}}{2} = \frac{10}{2}$

$\sqrt{5x} = 5$

Taking square on B.S

$\left(\sqrt{5x}\right)^2 = (5)^2$

$5x = 25$

Divide B.S by 5

$\frac{5x}{5} = \frac{25}{5}$

$x = 5$

**Verification**

Put $x = 5$ in equ (i)

$2\sqrt{5(5)} - 3 = 7$

$2\sqrt{25} - 3 = 7$

$2(5) - 3 = 7$

$10 - 3 = 7$

$7 = 7$

Thus Solution Set $= \{5\}$

7. $\mathbf{\sqrt{2x - 7} + 8 = 11}$

**Solution:**

$\sqrt{2x - 7} + 8 = 11 \dots\dots equ(i)$

Subtract 8 from B.S

$\sqrt{2x - 7} + 8 - 8 = 11 - 8$

$\sqrt{2x - 7} = 3$

Taking square on B.S

$\left(\sqrt{2x - 7}\right)^2 = (3)^2$

$2x - 7 = 9$

Add 7 on B.S

$2x - 7 + 7 = 9 + 7$

$2x = 16$

Divide B.S by 2

$\frac{2x}{2} = \frac{16}{2}$

$x = 8$

**Ex # 7.2**

**Verification**

Put $x = 8$ in equ (i)

$\sqrt{2(8) - 7} + 8 = 11$

$\sqrt{16 - 7} + 8 = 11$

$\sqrt{9} + 8 = 11$

$3 + 8 = 11$

$11 = 11$

Thus Solution Set $= \{8\}$

8. $\mathbf{22 = 17 + \sqrt{40 - 3y}}$

**Solution:**

$22 = 17 + \sqrt{40 - 3y} \dots\dots equ(i)$

Subtract 17 from B.S

$22 - 17 = 17 - 17 + \sqrt{40 - 3y}$

$5 = \sqrt{40 - 3y}$

$\sqrt{40 - 3y} = 5$

Taking square on B.S

$\left(\sqrt{40 - 3y}\right)^2 = (5)^2$

$40 - 3y = 25$

Subtract 40 from B.S

$40 - 40 - 3y = 25 - 40$

$-3y = -15$

$3y = 15$

Divide B.S by 3

$\frac{3y}{3} = \frac{15}{3}$

$y = 5$

**Verification**

Put $x = 5$ in equ (i)

$22 = 17 + \sqrt{40 - 3(5)}$

$22 = 17 + \sqrt{40 - 15}$

$22 = 17 + \sqrt{25}$

$22 = 17 + 5$

$22 = 22$

Thus Solution Set $= \{5\}$

**Ex # 7.3**

**Absolute value**

The absolute value of a number is always be non–negative.

**Example**

$$|5| = 5$$

And also

$$|-5| = 5$$

**Note:**

It should be noted that $|x|$ can never be negative, that is $|x| \geq 0$

$|0| = 0$

**Solution of Absolute value equation**

To solve equations involving absolute value in one variable, we have to consider both the possible values of the variable.

**Example**

$$|x| = 2$$

Then there is two possibilities

$$x = 2$$

Or

$$x = -2$$



**Example # 9**

$|\boldsymbol{x} - \mathbf{1}| = \mathbf{7}$

**Solution:**

$|x - 1| = 7$

*There are two possibilities*

*Either*

$x - 1 = 7 \;\; \dots\dots equ(i)$

*or*

$x - 1 = -7 \;\; \dots\dots equ(ii)$

$Now\ equ(i) \Longrightarrow$

$x - 1 = 7$

Add 1 on B.S

$x - 1 + 1 = 7 + 1$

$x = 8$

$Now\ equ(ii) \Longrightarrow$

$x - 1 = -7$

Add 1 on B.S

$x - 1 + 1 = -7 + 1$

$x = -6$

Solution Set $= \{8, -6\}$

**Ex # 7.3**

**Example # 10**

$|\mathbf{3}\boldsymbol{x} - \mathbf{5}| + \mathbf{7} = \mathbf{11}$

**Solution:**

$|3x - 5| + 7 = 11$

Subtract 7 from B.S

$|3x - 5| + 7 - 7 = 11 - 7$

$|3x - 5| = 4$

*There are two possibilities*

*Either*

$3x - 5 = 4 \;\; \dots\dots equ(i)$

*or*

$3x - 5 = -4 \;\; \dots\dots equ(ii)$

$Now\ equ(i) \Longrightarrow$

$3x - 5 = 4$

Add 5 on B.S

$3x - 5 + 5 = 4 + 5$

$3x = 9$

$Divide\ B.S\ by\ 3$

$\dfrac{3x}{3} = \dfrac{9}{3}$

$x = 3$

$Now\ equ(ii) \Longrightarrow$

$3x - 5 = -4$

Add 5 on B.S

$3x - 5 + 5 = -4 + 5$

$3x = 1$

$Divide\ B.S\ by\ 3$

$\dfrac{3x}{3} = \dfrac{1}{3}$

$x = \dfrac{1}{3}$

Solution Set $= \left\{3, \dfrac{1}{3}\right\}$

# Exercise # 7.3

**Page # 182**

**Q: 1.** **Solve for $\boldsymbol{x}$**

$|\boldsymbol{x} + \mathbf{3}| = \mathbf{5}$

**Solution:**

$|x + 3| = 5$

*There are two possibilities*

*Either*

$x + 3 = 5 \;\; \dots\dots equ(i)$

*or*

$x + 3 = -5 \;\; \dots\dots equ(ii)$

# Chapter # 7

**Ex # 7.3**

*Now* $equ(i) \Longrightarrow$

$x+3=5$

Subtract 3 from B.S

$x+3-3-=5-3$

$x=2$

*Now* $equ(ii) \Longrightarrow$

$x+3=-5$

Subtract 3 from B.S

$x+3-3-=-5-3$

$x=-8$

Solution Set $=\{2,-8\}$

**2.** $\boldsymbol{|-5x+1|=6}$

**Solution:**

$|-5x+1|=6$

*There are two possibilities*

*Either*

$-5x+1=6 \;\ldots\ldots equ(i)$

*or*

$-5x+1=-6 \;\ldots\ldots equ(ii)$

*Now* $equ(i) \Longrightarrow$

$-5x+1=6$

Subtract 1 from B.S

$-5x+1-1=6-1$

$-5x=5$

*Divide B.S by* $-5$

$\frac{-5x}{-5}=\frac{5}{-5}$

$x=-1$

*Now* $equ(ii) \Longrightarrow$

$-5x+1=-6$

Subtract 1 from B.S

$-5x+1-1=-6-1$

$-5x=-7$

$5x=7$

*Divide B.S by* $5$

$\frac{5x}{5}=\frac{7}{5}$

$x=\frac{7}{5}$

Solution Set $=\left\{-1,\frac{7}{5}\right\}$

**Ex # 7.3**

**3.** $\boldsymbol{\left|\frac{3}{4}x-8\right|=1}$

**Solution:**

$\left|\frac{3}{4}x-8\right|=1$

*There are two possibilities*

*Either*

$\frac{3}{4}x-8=1 \;\ldots\ldots equ(i)$

*or*

$\frac{3}{4}x-8=-16 \;\ldots\ldots equ(ii)$

*Now* $equ(i) \Longrightarrow$

$\frac{3}{4}x-8=1$

Add 8 on B.S

$\frac{3}{4}x-8+8=1+8$

$\frac{3}{4}x=9$

*Multiply B.S by* $\frac{4}{3}$

$\frac{4}{3}\times\frac{3}{4}x=\frac{4}{3}\times 9$

$x=4\times 3$

$x=12$

*Now* $equ(ii) \Longrightarrow$

$\frac{3}{4}x-8=-1$

Add 8 on B.S

$\frac{3}{4}x-8+8=-1+8$

$\frac{3}{4}x=7$

*Multiply B.S by* $\frac{4}{3}$

$\frac{4}{3}\times\frac{3}{4}x=\frac{4}{3}\times 7$

$x=\frac{28}{3}$

Solution Set $=\left\{12,\frac{28}{3}\right\}$

# Chapter # 7

**Ex # 7.3**

**4.** $|\boldsymbol{x-4}| = \mathbf{3}$

**Solution:**

$|x-4| = 3$

*There are two possibilities*

*Either*

$x - 4 = 3 \;\dots\dots equ(i)$

*or*

$x - 4 = -3 \;\dots\dots equ(ii)$

*Now* $equ(i) \Longrightarrow$

$x - 4 = 3$

Add 4 on B.S

$x - 4 + 4 = 3 + 4$

$x = 7$

*Now* $equ(ii) \Longrightarrow$

$x - 4 = -3$

Add 4 on B.S

$x - 4 + 4 = -3 + 4$

$x = 1$

Solution Set $= \{7, 1\}$

**5.** $|\boldsymbol{3x+4}| = \mathbf{-2}$

**Solution:**

$|3x + 4| = -2$

As there is no such a number whose absolute value is negative

Thus Solution Set $= \{\ \}$

**6.** $|\boldsymbol{2x-9}| = \mathbf{0}$

**Solution:**

$|2x - 9| = 0$

$|x| = 0 \Longrightarrow x = 0$

So

$2x - 9 = 0$

Add 9 on B.S

$2x - 9 + 9 = 0 + 9$

$2x = 9$

*Divide B.S by 2*

$\frac{2x}{2} = \frac{9}{2}$

$x = \frac{9}{2}$

Solution Set $= \left\{\frac{9}{2}\right\}$

**Ex # 7.3**

**7.** $\left|\boldsymbol{\frac{3x-2}{5}}\right| = \mathbf{7}$

**Solution:**

$\left|\frac{3x-2}{5}\right| = 7$

*There are two possibilities*

*Either*

$\frac{3x-2}{5} = 7 \;\dots\dots equ(i)$

*or*

$\frac{3x-2}{5} = -7 \;\dots\dots equ(ii)$

*Now* $equ(i) \Longrightarrow$

$\frac{3x-2}{5} = 7$

Multiply B.S by 5

$5 \times \frac{3x-2}{5} = 5 \times 7$

$3x - 2 = 35$

Add 2 on B.S

$3x - 2 + 2 = 35 + 2$

$3x = 37$

*Divide B.S by 3*

$\frac{3x}{3} = \frac{37}{3}$

$x = \frac{37}{3}$

*Now* $equ(ii) \Longrightarrow$

$\frac{3x-2}{5} = -7$

Multiply B.S by 5

$5 \times \frac{3x-2}{5} = -7 \times 5$

$3x - 2 = -35$

Add 2 on B.S

$3x - 2 + 2 = -35 + 2$

$3x = -33$

*Divide B.S by 3*

$\frac{3x}{3} = \frac{-33}{3}$

$x = -11$

Solution Set $= \left\{\frac{37}{3}, -11\right\}$

**Ex # 7.3**

**8.** $\mathbf{4|5x-2|+3=11}$

**Solution:**

$4|5x-2|+3=11$

Subtract 3 from B.S

$4|5x-2|+3-3=11-3$

$4|5x-2|=8$

Divide B.S by 4

$\frac{4|5x-2|}{4}=\frac{8}{4}$

$|5x-2|=2$

$There\ are\ two\ possibilities$

$Either$

$5x-2=2\ \ \dots\dots equ(i)$

$or$

$5x-2=-2\ \ \dots\dots equ(ii)$

$Now\ equ(i)\Longrightarrow$

$5x-2=2$

Add 2 on B.S

$5x-2+2=2+2$

$5x=4$

$Divide\ B.S\ by\ 5$

$\frac{5x}{5}=\frac{4}{5}$

$x=\frac{4}{5}$

$Now\ equ(ii)\Longrightarrow$

$5x-2=2$

Add 2 on B.S

$5x-2+2=-2+2$

$5x=0$

$Divide\ B.S\ by\ 5$

$\frac{5x}{5}=\frac{0}{5}$

$x=0$

Solution Set $=\left\{\frac{4}{5},0\right\}$

**9.** $\mathbf{\frac{2}{5}|4x-3|-9=-1}$

**Solution:**

$\frac{2}{5}|4x-3|-9=-1$

Add 9 on B.S

$\frac{2}{5}|4x-3|-9+9=-1+9$

**Ex # 7.3**

$\frac{2}{5}|4x-3|=8$

$Multiply\ B.S\ by\ \frac{5}{2}$

$\frac{5}{2}\times\frac{2}{5}|4x-3|=\frac{5}{2}\times 8$

$|4x-3|=5\times 4$

$|4x-3|=20$

$There\ are\ two\ possibilities$

$Either$

$4x-3=20\ \ \dots\dots equ(i)$

$or$

$4x-3=-20\ \ \dots\dots equ(ii)$

$Now\ equ(i)\Longrightarrow$

$4x-3=20$

Add 3 on B.S

$4x-3+3=20+3$

$4x=23$

$Divide\ B.S\ by\ 4$

$\frac{4x}{4}=\frac{23}{4}$

$x=\frac{23}{4}$

$Now\ equ(ii)\Longrightarrow$

$4x-3=-20$

Add 3 on B.S

$4x-3+3=-20+3$

$4x=-17$

$Divide\ B.S\ by\ 4$

$\frac{4x}{4}=\frac{-17}{4}$

$x=\frac{-17}{4}$

Solution Set $=\left\{\frac{23}{4},\frac{-17}{4}\right\}$

# Chapter # 7

## Ex # 7.4

**Linear Inequality**

**Inequality**

The relation which compares two real numbers e.g. $x$ and $y$ but $x \neq y$.

Following symbols of inequality as under:

$<$ less than

$>$ greater than

$\leq$ less than or equal to

$\geq$ greater than or equal to

**We have the following possibilities**

$x < y$ *means that* $x$ *less than* $y$

$x > y$ *means that* $x$ *greater than* $y$

$x \leq y$ *means that* $x$ *less than or equal to* $y$

$x \geq y$ *means that* $x$ *is grater than or equal to* $y$

**Solution of Linear Inequalities**

The set of all possible values of the variable which makes the inequality a true statement is called solution set of the inequality.

It is simple to represent the solution of an inequality with the help on real number line.

**Real Number Line**

A line whose points are represented by real number is called real number line.

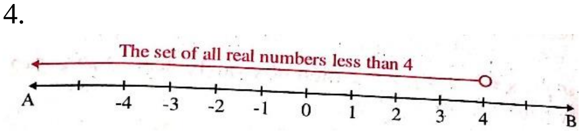
**Geometrical representation with examples**

**Example:** $\boldsymbol{x < 4}$

$x < 4$, it means that all real numbers less than 4.

Geometrically all real numbers lying to the left of 4 but 4 is not included.

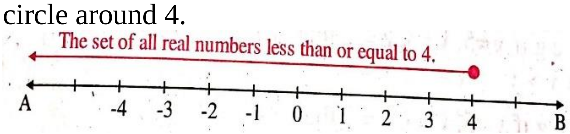
This is represented by using hollow circle around 4.




**Example:** $\boldsymbol{x \leq 4}$

$x \leq 4$, it means that all real numbers less than or equal to 4. Geometrically all real numbers lying to the left of 4 and also including 4.

This is represented by using thick, filled or solid circle around 4.




## Ex # 7.4

**Example # 11**

**Show $-2 < x < 5$ on a number line.**

**Solution:**

$$-2 < x < 5$$

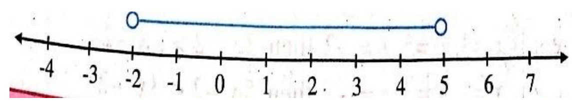
$-2 < x < 5$ means the set of real numbers which are greater than $-2$ but less than 5.

$-2 < x < 5$ means the set of real numbers which are between $-2$ and 5

Geometrical $-2 < x < 5$ means the set of real numbers lying to the right of $-2$ and left to 5.

**Note:**

Here $-2$ and 5 are not included.




**Properties of Inequality of Real Numbers**

**Trichotomy Property**

Trichotomy property means when comparing two numbers, one of the following must be true:

(a) $a = b$

(b) $a < b$

(c) $a > b$

**Examples:**

(i) $5 = 5$

(ii) $3 < 5$

(iii) $3 > 5$

**Transitive Property**

(a) If $a > b$ and $b > c$ then $a > c$

**Example:**

If $7 > 5$ and $5 > 3$ then $\mathbf{7} > 3$

(b) If $a < b$ and $b < c$ then $a < c$

**Example:**

If $3 < 5$ and $5 < 7$ then $\mathbf{3} < 7$

**Additive Property**

(a) If $a < b$ then $a + c < b + c$

(b) If $a < b$ then $a - c < b - c$

**Examples:**

(i) $3 < 5$ then $3 + 2 < 5 + 2$

(ii) $3 < 5$ then $3 - 2 < 5 - 2$

(iii) $\boldsymbol{x - 3 > 5}$

Add 3 on B.S

$x - 3 + 3 = 5 + 3$

$x = 8$

# Chapter # 7



## Ex # 7.4

(c) If $a > b$ then $a + c > b + c$
(d) If $a > b$ then $a - c > b - c$

**Example:**

(i) $5 > 3$ then $5 + 2 > 3 + 2$
(ii) $5 > 3$ then $5 - 7 > 3 - 7$ So $-2 > -4$
(iii) $\boldsymbol{x + 3 > 5}$

Subtract 3 from B.S

$x + 3 - 3 = 5 - 3$

$x = 2$

**Multiplicative Property**

**1. When $c > 0$:**

(a) If $a < b$ then $ac < bc$
(b) If $a > b$ then $ac > bc$

**Example:**

(i) $5 > 3$ then $5 \times 2 > 3 \times 2$
(ii) $\boldsymbol{\frac{x}{3} > 5}$

Multiply B.S by 3

$\frac{x}{3} \times 3 > 5 \times 3$

$x > 15$

(iii) $\boldsymbol{2x > 24}$

$\frac{2x}{2} > \frac{24}{2}$

Divide B.S by 2

$x > 12$

**2. When $c < 0$:**

(a) If $a < b$ then $ac > bc$
(b) If $a > b$ then $ac < bc$

**Example:**

(i) $5 > 3$ then $5 \times -2 < 3 \times -2$ So $-10 < -6$
(ii) $\frac{x}{-3} < 5$

Multiply B.S by $-3$

$\frac{x}{-3} \times -3 > 5 \times -3$

$x > -15$

**Example # 12**

**Write the names of properties used in the following statements.**

$21 < 31 \Rightarrow 31 < 41$

$21 < 31 \Rightarrow 21 + 10 < 31 + 10$

Hence $21 < 31 \Rightarrow 31 < 41$

Additive Property

## Ex # 7.4

$\boldsymbol{15 > 8 \Rightarrow 22 > 15}$

**Solution:**

$15 > 8 \Rightarrow 15 + 7 > 8 + 7$

Hence $15 > 8 \Rightarrow 22 > 15$

Additive Property

$\boldsymbol{10 < 20 \Rightarrow 30 < 60}$

**Solution:**

$10 < 20 \Rightarrow 10 \times 3 < 20 \times 3$

Hence $10 < 20 \Rightarrow 30 < 60$

Multiplicative Property

$\boldsymbol{-12 > -15 \Rightarrow 24 < 30}$

**Solution:**

$-12 > -15 \Rightarrow -12 \times -2 < -15 \times -2$

Hence $-12 > -15 \Rightarrow 24 < 30$

Multiplicative Property

**If $x > 4$ and $4 > z$ then $x > z$**

**Solution:**

$x > 4$ and $4 > z \Rightarrow x > z$

Transitive Property

**Solution of Linear Inequalities**

Linear inequalities are solved in almost the same way as linear equations.

**Principles in Inequalities**

(i) If $a > b$, then
$a + c > b + c$ , $a - c > b - c$ , $a - b > 0$

(ii) If $a > b$ and $k > 0$, then
$ka > kb$ and $\frac{a}{k} > \frac{b}{k}$

(iii) If $a > b$ and $k < 0$, then
$ka < kb$ and $\frac{a}{k} < \frac{b}{k}$

**Example # 13**

**You are checking a bag at an airport. Bags can weigh no more than 50 Kgs. Your bag weighs 16.8 kg. Find the possible weight w (in Kg) that you can add to the bag.**

**Solution:**

$Bag's\ weight + weight\ you\ can\ add \leq weight\ limit$

$16.8 + W \leq 50$

Subtract 16.8 from B.S

$16.8 - 16.8 + W \leq 50 - 16.8$

$W \leq 33.2$

So we can add upto 33.2 Kg

# Chapter # 7

**Ex # 7.4**

**<u>Example # 14</u> (i)**

***Solve the inequality*** $\boldsymbol{2\left(\frac{x}{4}+1\right)<\frac{3}{2}}$

***where x is a natural number***

**<u>Solution</u>:**

$2\left(\frac{x}{4}+1\right)<\frac{3}{2}$

$2\left(\frac{x+4}{4}\right)<\frac{3}{2}$

$\frac{x+4}{2}<\frac{3}{2}$

Multiply B.S by 2

$2\times\frac{x+4}{2}<2\times\frac{3}{2}$

$x+4<3$

Subtract 4 from B.S

$x+4-4<3-4$

$x<-1$

*As natural number cannot be less than* $-1$,
*then it has no solution*

Thus, Solution Set $=\{\ \}$

**<u>Example # 14</u> (ii)**

***Solve the inequality*** $\boldsymbol{2\left(\frac{x}{4}+1\right)<\frac{3}{2}}$

***where x is a real number***

**<u>Solution</u>:**

$2\left(\frac{x}{4}+1\right)<\frac{3}{2}$

$2\left(\frac{x+4}{4}\right)<\frac{3}{2}$

$\frac{x+4}{2}<\frac{3}{2}$

Multiply B.S by 2

$2\times\frac{x+4}{2}<2\times\frac{3}{2}$

$x+4<3$

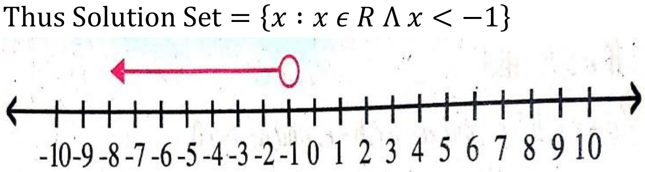
Subtract 4 from B.S

$x+4-4<3-4$

$x<-1$

Thus it consists of all real numbers less than $-1$

Thus Solution Set $=\{x : x \in R \wedge x<-1\}$




**Ex # 7.4**

**<u>Example # 15</u> (i)**

***Solve the inequality*** $\boldsymbol{x-\frac{5}{7}\leq\frac{15+2x}{7}}$

***where x is a natural number***

**<u>Solution</u>:**

$x-\frac{5}{7}\leq\frac{15+2x}{7}$

$\frac{7x-5}{7}\leq\frac{15+2x}{7}$

Multiply B.S by 7

$7\times\frac{7x-5}{7}\leq 7\times\frac{15+2x}{7}$

$7x-5\leq 15+2x$

Add 5 on B.S

$7x-5+5\leq 15+5+2x$

$7x\leq 20+2x$

Subtract $2x$ from B.S

$7x-2x\leq 20+2x-2x$

$5x\leq 20$

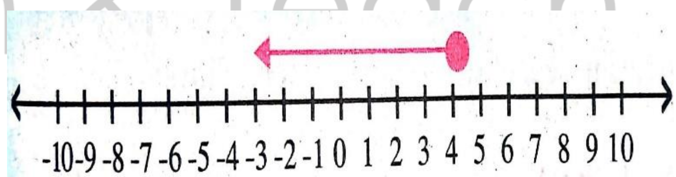
Divide B.S by 5

$\frac{5x}{5}\leq\frac{20}{5}$

$x\leq 4$

*As x is natural number and less than or equal to* 4

Thus Solution Set $=\{1,2,3,4\}$




**<u>Example # 15</u> (ii)**

***Solve the inequality*** $\boldsymbol{x-\frac{5}{7}\leq\frac{15+2x}{7}}$

***where x is a real number***

**<u>Solution</u>:**

$x-\frac{5}{7}\leq\frac{15+2x}{7}$

$\frac{7x-5}{7}\leq\frac{15+2x}{7}$

Multiply B.S by 7

$7\times\frac{7x-5}{7}\leq 7\times\frac{15+2x}{7}$

$7x-5\leq 15+2x$

Add 5 on B.S

$7x-5+5\leq 15+5+2x$

$7x\leq 20+2x$

Ex # 7.4

Subtract $2x$ from B.S

$7x - 2x \leq 20 + 2x - 2x$

$5x \leq 20$

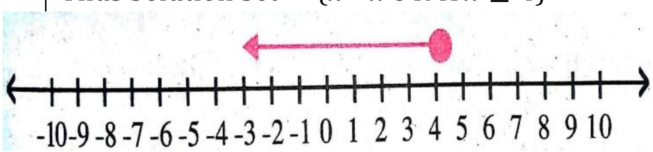
Divide B.S by 5

$\frac{5x}{5} \leq \frac{20}{5}$

$x \leq 4$

*Thus it consists of all real numbers less than or equal to 4*

Thus Solution Set $= \{x : x \in R \wedge x \leq 4\}$




**Example # 16**

***Solve the inequality*** $\frac{x+3}{2} \leq \frac{x-5}{3}$ ***where*** $x \in R$

**Solution:**

$\frac{x+3}{2} \leq \frac{x-5}{3}$

Multiply B.S by 6

$6 \times \frac{x+3}{2} \leq 6 \times \frac{x-5}{3}$

$3(x+3) \leq 2(x-5)$

$3x + 9 \leq 2x - 10$

Subtract 9 from B.S

$3x + 9 - 9 \leq 2x - 10 - 9$

$3x \leq 2x - 19$

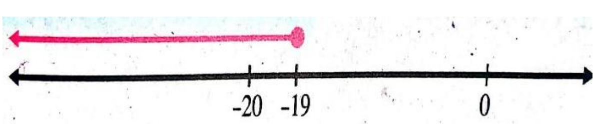
Subtract $2x$ from B.S

$3x - 2x \leq 2x - 2x - 19$

$x \leq -19$

*Thus it consists of all real numbers less than or equal to* $-19$

Thus Solution Set $= \{x : x \in R \wedge x \leq -19\}$




# Exercise # 7.4

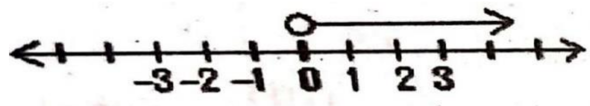
**Page # 188**

**Q1: Show the following inequalities on number line.**

**(i)** $x > 0$

**Solution:**

$x > 0$




**(ii)** $x < 0$

**Solution:**

$x < 0$


**(iii)** $\frac{x-3}{2} \leq -1$

**Solution:**

$\frac{x-3}{2} \leq -1$

Multiply B.S by 2

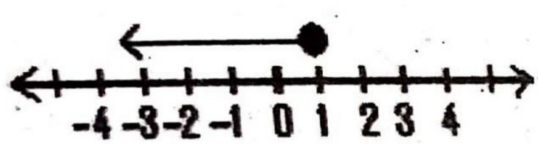
$2 \times \frac{x-3}{2} \leq -1 \times 2$

$x - 3 \leq -2$

Add 3 on B.S

$x - 3 + 3 \leq -2 + 3$

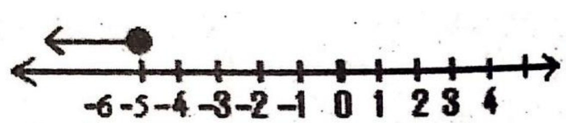
$x \leq 1$




**(v)** $x \leq -5$

**Solution:**

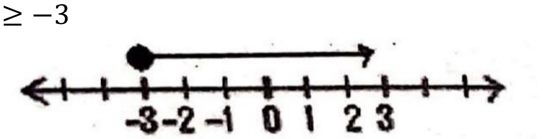
$x \leq -5$




$x \geq -3$

**Solution:**

$x \geq -3$



# Chapter # 7

## Ex # 7.4

(vi) $\dfrac{3x-2}{6} > \dfrac{5}{2}$

**Solution**:

$\dfrac{3x-2}{6} > \dfrac{5}{2}$

Multiply B.S by 6

$6 \times \dfrac{3x-2}{6} > 6 \times \dfrac{5}{2}$

$3x - 2 > 3 \times 5$

$3x - 2 > 15$

Add 2 on B.S

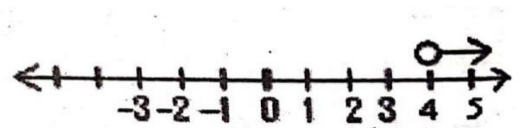
$3x - 2 + 2 > 15 + 2$

$3x > 17$

Divide B.S by 3

$\dfrac{3x}{3} > \dfrac{17}{3}$

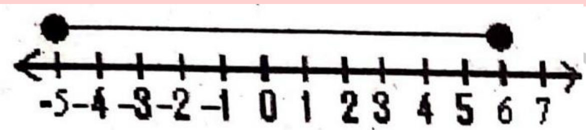
$x > 5.67$




(vii) $-5 \le x \le 6$

**Solution**:

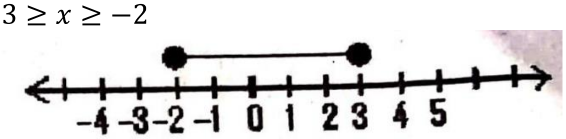
$-5 \le x \le 6$




(viii) $3 \ge x \ge -2$

**Solution**:

$3 \ge x \ge -2$




(ix) $0 < \dfrac{x}{4} - 1 < \dfrac{1}{2}$

**Solution**:

$0 < \dfrac{x}{4} - 1 < \dfrac{1}{2}$

Multiply by 4

$4 \times 0 < 4\left(\dfrac{x}{4} - 1\right) < 4 \times \dfrac{1}{2}$

$0 < 4 \times \dfrac{x}{4} - 4 \times 1 < 2 \times 1$

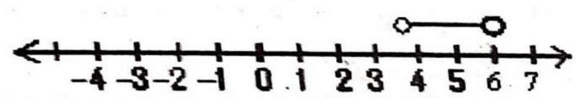
$0 < x - 4 < 2$

## Ex # 7.4

Add 4

$0 + 4 < x - 4 + 4 < 2 + 4$

$4 < x < 6$




(x) $0 < \dfrac{x+3}{2} < \dfrac{3}{2}$

**Solution**:

$0 < \dfrac{x+3}{2} < \dfrac{3}{2}$

Multiply by 2

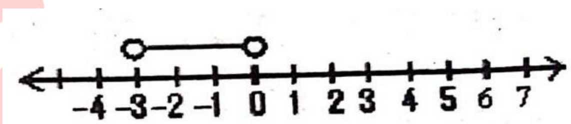
$2 \times 0 < 2 \times \dfrac{x+3}{2} < 2 \times \dfrac{3}{2}$

$0 < x + 3 < 3$

Subtract 3

$0 - 3 < x + 3 - 3 < 3 - 3$

$-3 < x < 0$




**Q2: Find the solution set of the following inequalities.**

(i) $7 - 2x \ge 1, \quad x \in N$

**Solution**:

$7 - 2x \ge 1, \quad x \in N$

Now

$7 - 2x \ge 1$

Subtract 7 from B.S

$7 - 7 - 2x \ge 1 - 7$

$-2x \ge -6$

Divide B.S by $-2$

$\dfrac{-2x}{-2} \le \dfrac{-6}{-2}$

$x \le 3$

*As* $x \in N$ *and* $x \le 3$

Thus Solution Set $= \{1, 2, 3\}$

(ii) $5x + 4 < 34, \quad x \in N$

**Solution**:

$5x + 4 < 34, \quad x \in N$

Now

$5x + 4 < 34$

Subtract 4 from B.S

$5x + 4 - 4 < 34 - 4$

$5x < 30$

# Chapter # 7

## Ex # 7.4

Divide B.S by 5

$$\frac{5x}{5} < \frac{30}{5}$$

$x < 6$

*As* $x \in N$ *and* $x < 6$

Thus Solution Set $= \{1, 2, 3, 4, 5\}$

**(iii)** $\boldsymbol{\frac{8x+1}{2} < 2x - 1.5\,, \quad x \in R}$

**Solution:**

$$\frac{8x+1}{2} < 2x - 1.5\,, \quad x \in R$$

Now

$$\frac{8x+1}{2} < 2x - 1.5$$

Multiply B.S by 2

$$2 \times \frac{8x+1}{2} < 2(2x - 1.5)$$

$8x + 1 < 4x - 3$

Now

$8x - 4x < -3 - 1$

$4x < -4$

Divide B.S by 4

$$\frac{4x}{4} < \frac{-4}{4}$$

$x < -1$

*As* $x \in R$ *and* $x < -1$

Thus Solution Set $= \{x : x \in R \wedge x < -1\}$

**(iv)** $\boldsymbol{(4x+3) \geq 23\,, \quad x \in \{1, 2, 3, 4, 5, 6\}}$

**Solution:**

$(4x+3) \geq 23\,, \quad x \in \{1, 2, 3, 4, 5, 6\}$

**Now**

$4x + 3 \geq 23$

Subtract 3 from B.S

$4x + 3 - 3 \geq 23 - 3$

$4x \geq 20$

Divide B.S by 4

$$\frac{4x}{4} \geq \frac{20}{4}$$

$x \geq 5$

*As* $x \in \{1, 2, 3, 4, 5, 6\}$ *and* $x \geq 5$

Thus Solution Set $= \{5, 6\}$

## Ex # 7.4

**(v)** $\boldsymbol{5x + 1 \geq 13 - x\,, \quad x \in \{-2, -1, 0, 1, 2, 3, 4, 5\}}$

**Solution:**

$5x + 1 \geq 13 - x\,, \quad x \in \{-2, -1, 0, 1, 2, 3, 4, 5\}$

Now

$5x + 1 \geq 13 - x$

Now

$5x + x \geq 13 - 1$

$6x \geq 12$

Divide B.S by 6

$$\frac{6x}{6} \geq \frac{12}{6}$$

$x \geq 2$

*As* $x \in \{-2, -1, 0, 1, 2, 3, 4, 5\}$ *and* $x \geq 2$

Thus Solution Set $= \{2, 3, 4, 5\}$

**(vi)** $\boldsymbol{\frac{2x+6}{2} \leq \frac{x-9}{3}\,, \quad x \in R}$

**Solution:**

$$\frac{2x+6}{2} \leq \frac{x-9}{3}\,, \quad x \in R$$

Now

$$\frac{2x+6}{2} \leq \frac{x-9}{3}$$

Multiply B.S by 6

$$6 \times \frac{2x+6}{2} \leq 6 \times \frac{x-9}{3}$$

$3(2x+6) \leq 2(x-9)$

$6x + 18 \leq 2x - 18$

Now

$6x - 2x \leq -18 - 18$

$4x \leq -36$

Divide B.S by 4

$$\frac{4x}{4} \leq \frac{-36}{4}$$

$x \leq -9$

*As* $x \in R$ *and* $x \leq -9$

Thus Solution Set $= \{x : x \in R \wedge x \leq -9\}$

**(vii)** $\boldsymbol{\frac{x-1}{3} \leq \frac{1-x}{2}\,, \quad x \in Z}$

**Solution:**

$$\frac{x-1}{3} \leq \frac{1-x}{2}\,, \quad x \in Z$$

Now

$$\frac{x-1}{3} \leq \frac{1-x}{2}$$

# Chapter # 7

## Ex # 7.4

Multiply B.S by 6

$6 \times \frac{x-1}{3} \leq 6 \times \frac{1-x}{2}$

$2(x-1) \leq 3(1-x)$

$2x - 2 \leq 3 - 3x$

Now

$2x + 3x \leq 3 + 2$

$5x \leq 5$

Divide B.S by 5

$\frac{5x}{5} \leq \frac{5}{5}$

$x \leq 1$

$As\ \ x \in Z\ and\ x \leq 1$

Thus Solution Set $= \{1, 0, -1, -2, -3 \dots \dots\}$

**Q3:** **Solve the following inequalities and plot the solution on the number line.**

**(i)** $\boldsymbol{\frac{x}{12} \leq \frac{1}{4}}$

**Solution:**

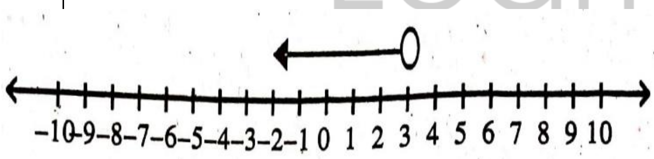
$\frac{x}{12} \leq \frac{1}{4}$

Multiply B.S by 12

$12 \times \frac{x}{12} \leq 12 \times \frac{1}{4}$

$x \leq 3 \times 1$

$x \leq 3$




**(ii)** $\boldsymbol{x + 7 \geq 2}$

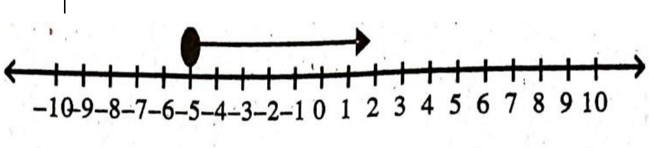
**Solution:**

$x + 7 \geq 2$

Subtract 7 from B.S

$x + 7 - 7 \geq 2 - 7$

$x \geq -5$




## Ex # 7.4

**(iii)** $\boldsymbol{3(x-2) > 15}$

**Solution:**

$3(x-2) > 15$

$3x - 6 > 15$

Add 6 on B.S

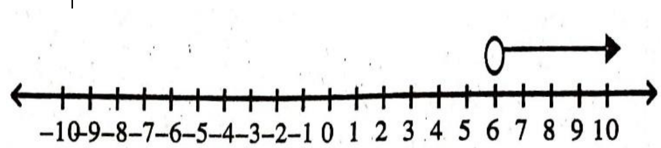
$3x - 6 + 6 > 15 + 6$

$3x > 21$

Divide B.S by 3

$\frac{3x}{3} > \frac{21}{3}$

$x > 7$




**(iv)** $\boldsymbol{\frac{1}{2} > \frac{x}{4} > -2}$

**Solution:**

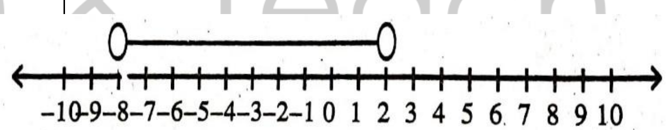
$\frac{1}{2} > \frac{x}{4} > -2$

Multiply by 4

$4 \times \frac{1}{2} > 4 \times \frac{x}{4} > -2 \times 4$

$2 \times 1 > x > -8$

$2 > x > -8$




**(v)** $\boldsymbol{2.5 \leq \frac{x}{2} + 1 \leq 4.5}$

**Solution:**

$2.5 \leq \frac{x}{2} + 1 \leq 4.5$

Multiply B.S by 2

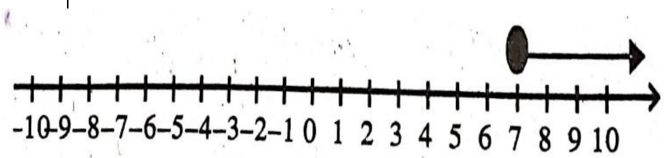
$2 \times 2.5 \leq 2\left(\frac{x}{2} + 1\right) \leq 2 \times 4.5$

$5 \leq x + 2 \leq 9$

Subtract 2 from them

$5 - 2 \leq x + 2 - 2 \leq 9 - 2$

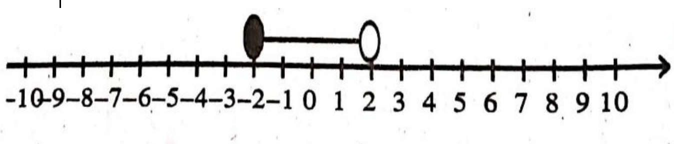
$3 \leq x \leq 7$



(vi) $-2 \le x < 2$

**Solution:**

$-2 \le x < 2$




# Review Ex # 7

**Page # 190**

**Q2:** **Solve the following equation for $x$**

**(i)** $5(3x+1) = 2(x-4)$

**Solution:**

$5(3x+1) = 2(x-4) \ldots \ldots equ(i)$

$15x + 5 = 2x - 8$

Subtract 5 from B.S

$15x + 5 - 5 = 2x - 8 - 5$

$15x = 2x - 13$

Subtract $2x$ from B.S

$15x - 2x = 2x - 2x - 13$

$13x = -13$

Divide B.S by 13

$\frac{13x}{13} = \frac{-13}{13}$

$x = -1$

**Verification**

Put $x = -1$ in equ (i)

$5(3(-1)+1) = 2(-1-4)$

$5(-3+1) = 2(-5)$

$5(-2) = -10$

$-10 = -10$

Solution Set $= \{-1\}$

**(ii)** $\frac{x-8}{3} + \frac{x-3}{2} = 0$

**Solution:**

$\frac{x-8}{3} + \frac{x-3}{2} = 0 \ldots \ldots equ(i)$

Multiply all terms by 6

$6 \times \frac{x-8}{3} + 6 \times \frac{x-3}{2} = 6 \times 0$

$2(x-8) + 3(x-3) = 0$

$2x - 16 + 3x - 9 = 0$

$2x + 3x - 16 - 9 = 0$

$5x - 25 = 0$

Add 25 on B.S



$5x - 25 + 25 = 0 + 25$

$5x = 25$

Divide B.S by 5

$\frac{5x}{5} = \frac{25}{5}$

$x = 5$

**Verification**

Put $x = 5$ in equ (i)

$\frac{5-8}{3} + \frac{5-3}{2} = 0$

$\frac{-3}{3} + \frac{2}{2} = 0$

$-1 + 1 = 0$

$0 = 0$

Solution Set $= \{5\}$

**(iii)** $\sqrt{2(5x-1)} = \sqrt{2x+14}$

**Solution:**

$\sqrt{2(5x-1)} = \sqrt{2x+14}$

$\sqrt{2(5x-1)} = \sqrt{2x+14} \ldots \ldots equ(i)$

Take square root on B.S

$\left(\sqrt{2(5x-1)}\right)^2 = \left(\sqrt{2x+14}\right)^2$

$2(5x-1) = 2x + 14$

$10x - 2 = 2x + 14$

Now

$10x - 2x = 14 + 2$

$8x = 16$

Divide B.S by 4

$\frac{8\sqrt{x}}{8} = \frac{16}{8}$

$\sqrt{x} = 2$

Taking square on B.S

$\left(\sqrt{x}\right)^2 = (2)^2$

$x = 4$

**Verification**

Put $x = 2$ in equ (i)

$\sqrt{2(5(2)-1)} = \sqrt{2(2)+14}$

$\sqrt{2(10-1)} = \sqrt{4+14}$

$\sqrt{2(9)} = \sqrt{18}$

$\sqrt{18} = \sqrt{18}$

$\sqrt{9 \times 2} = \sqrt{9 \times 2}$

$3\sqrt{2} = 3\sqrt{2}$

Solution Set $= \{36\}$

# Chapter # 7

**Review Ex # 7**

**(iv)** $|2x+7| = 9$

**Solution:**

$|2x+7| = 9$

*There are two possibilities*

*Either*

$2x + 7 = 9 \quad \ldots\ldots equ(i)$

*or*

$2x + 7 = -9 \quad \ldots\ldots equ(ii)$

$Now\ equ(i) \Longrightarrow$

$2x + 7 = 9$

Subtract 7 from B.S

$2x + 7 - 7 = 9 - 7$

$2x = 2$

Divide B.S by 2

$\frac{2x}{2} = \frac{2}{2}$

$x = 1$

$Now\ equ(ii) \Longrightarrow$

$2x + 7 = -9$

Subtract 7 from B.S

$2x + 7 - 7 = -9 - 7$

$2x = -16$

Divide B.S by 2

$\frac{2x}{2} = \frac{-16}{2}$

$x = -8$

$\text{Solution Set} = \{1, -8\}$

**Q3:** **Solve the following inequalities and graph the solution on the number line.**

**(i)** $-1 < \frac{x-3}{2} < 0$

**Solution:**

$-1 < \frac{x-3}{2} < 0$

Multiply by 2

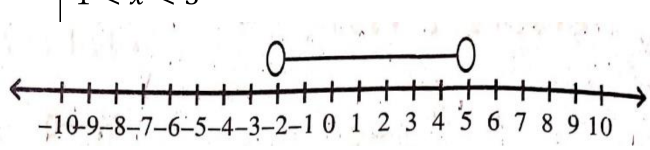
$-1 \times 2 < 2 \times \frac{x-3}{2} < 2 \times 0$

$-2 < x - 3 < 0$

Add 3

$-2 + 3 < x - 3 + 3 < 0 + 3$

$1 < x < 3$




**Review Ex # 7**

**(ii)** $-1 < \frac{x-4}{5} < 0$

**Solution:**

$-1 < \frac{x-4}{5} < 0$

Multiply by 5

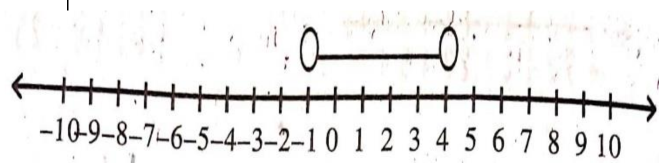
$-1 \times 5 < 5 \times \frac{x-4}{5} < 5 \times 0$

$-5 < x - 4 < 0$

Add 4

$-5 + 4 < x - 4 + 4 < 0 + 4$

$-1 < x < 4$




**(iii)** $7 < -3x + 1 \leq 13$

**Solution:**

$7 < -3x + 1 \leq 13$

Subtract 1

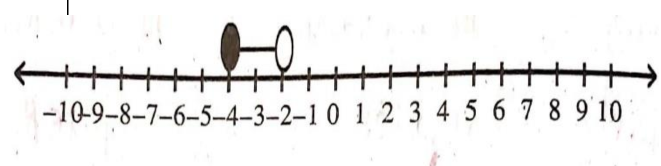
$7 - 1 < -3x + 1 - 1 \leq 13 - 1$

$6 < -3x \leq 12$

Divide B.S by 3

$\frac{6}{-3} > \frac{-3x}{-3} \geq \frac{12}{-3}$

$-2 > x \leq -4$



# Chapter # 7

## Review Ex # 7

**Q4:** **A father is 4 times older than his son. In 20 years, he will be twice as old as his son. What ages have they now?**

**Solution:**

Let the present age of son $= x$ years

So the present age of father $= 4x$ years

After twenty years

Age of son $= (x + 20)years$

and age of son $= (4x + 20)years$

According to condition

Age of father $= 2$(Age of son)

$4x + 20 = 2(x + 20)$

$4x + 20 = 2x + 40$

Now shift the variable and constant

$4x - 2x = 40 - 20$

$2x = 20$

Divide B.S by 2

$\frac{2x}{2} = \frac{20}{2}$

$x = 10$

Thus present age of son $= x = 10\ years$

And present age of father $= 4x\ years$

$= 4 \times 10\ years$

$= 40\ years$